Institute for Transport Studies

EARTH & ENVIRONMENT

UNIVERSITY OF LEEDS

Transport + Network Science
= Transport Network Science?

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Research in collaboration with

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David Watling [Professor]

Is Network Structure Important?
Some Questions…

What structure [topology] do these networks have? [*taxonomy*?]

Is network structure/topology important? [*When/where?*]

Is it meaningful to compare different networks, and their performance? [*Could we make it meaningful*?]

Can we understand anything about transferability e.g. of a transport policy? [*If policy A works in city X will it work in city Y?*]

Are there common emergent properties arising from network structure? [*Would this allow representation of detailed networks in an aggregate way?*]

[redacted]

In the 1960s there was an interest in considering the properties of “idealized” networks…

Street Routing Topologies: [Holroyd 1966]

$$Routing\ Factor = \frac{Network\ Distance}{Direct\ Distance}$$

Compare for theoretical city networks…

Ignores congestion, which would need demand location.

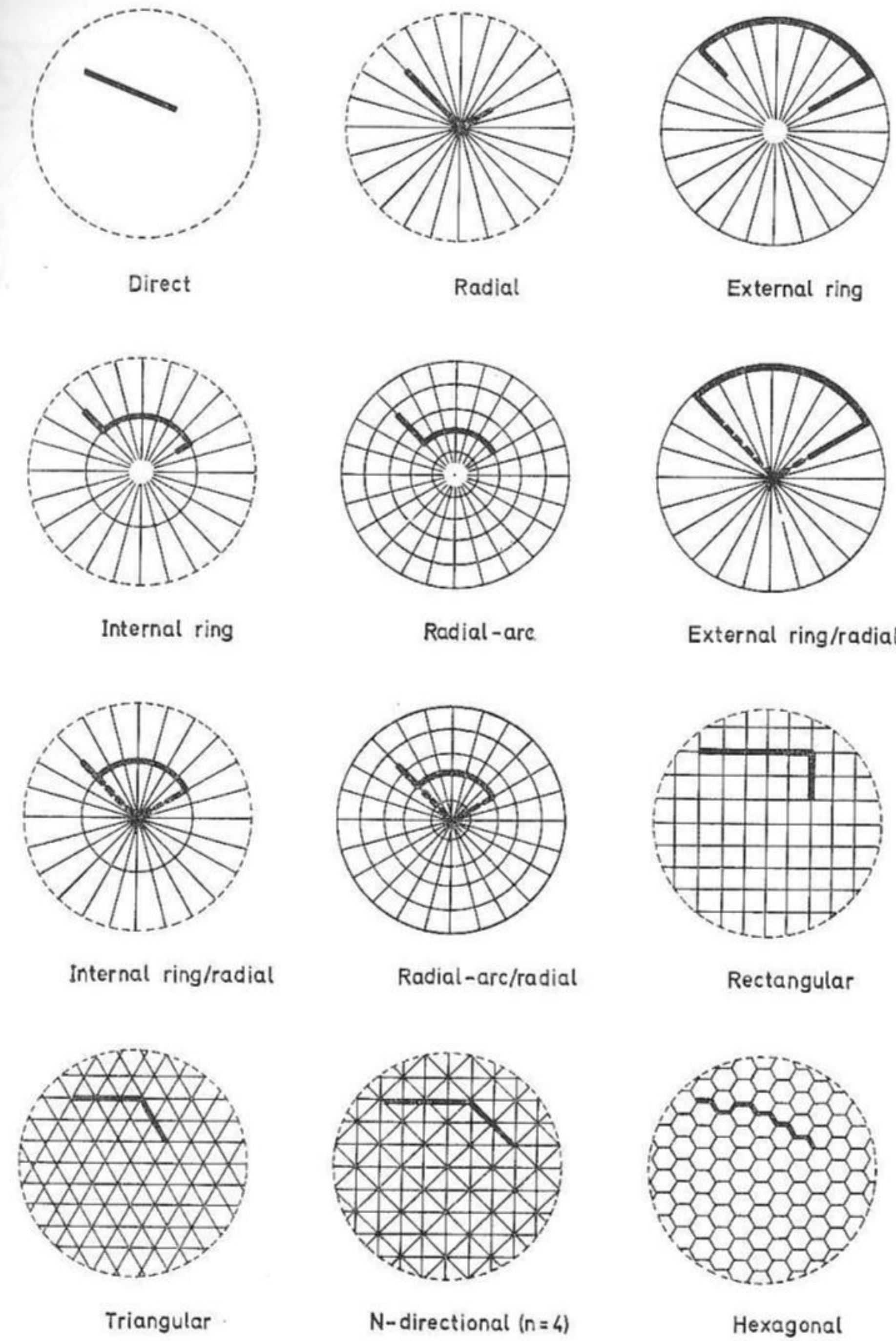


Fig. 3.10. Alternative routing systems within circular regions. Source: Holroyd, 1966, p. 33.

Topology & Routing Factor: [Smeed 1968]

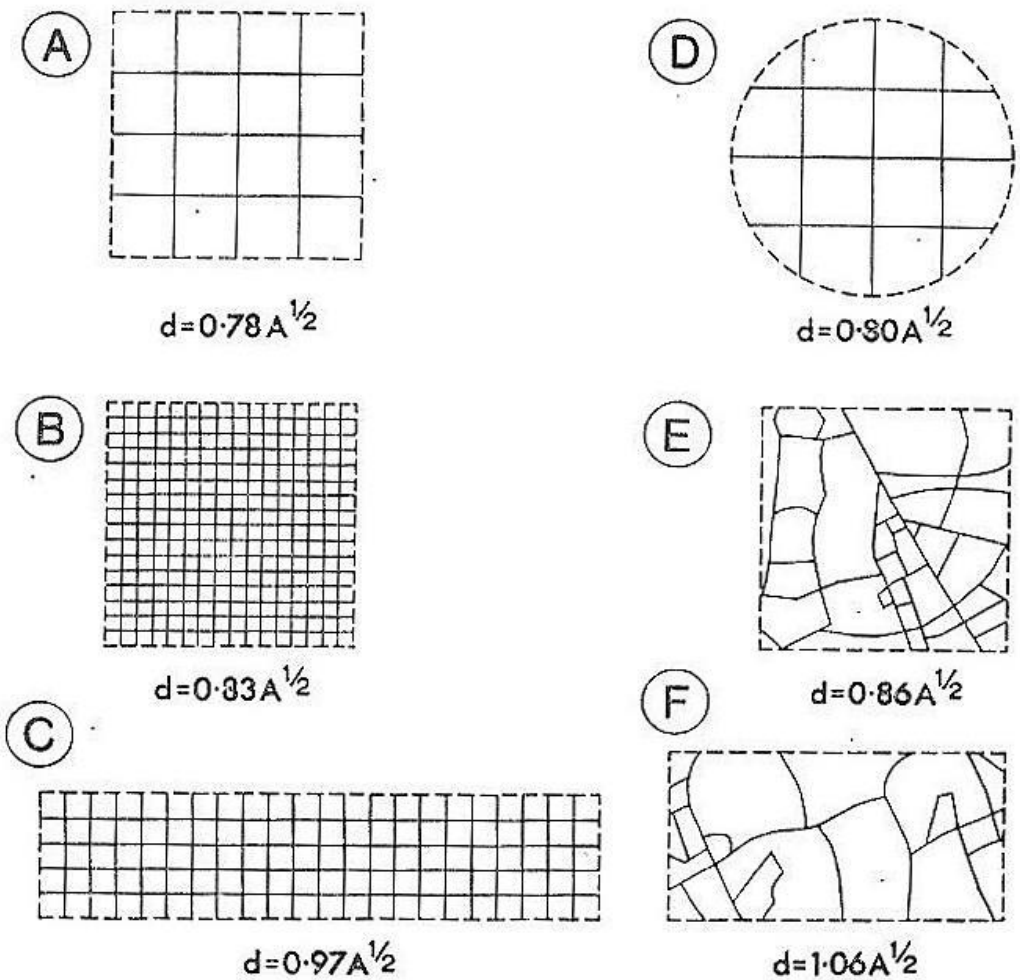


Fig. 3.7. Average distance travelled (d) on some imaginary and real road networks. A = area within boundary (broken line). Source: Smeed, 1968, pp. 5, 23.

Theoretical Network Topologies [1850-1963]

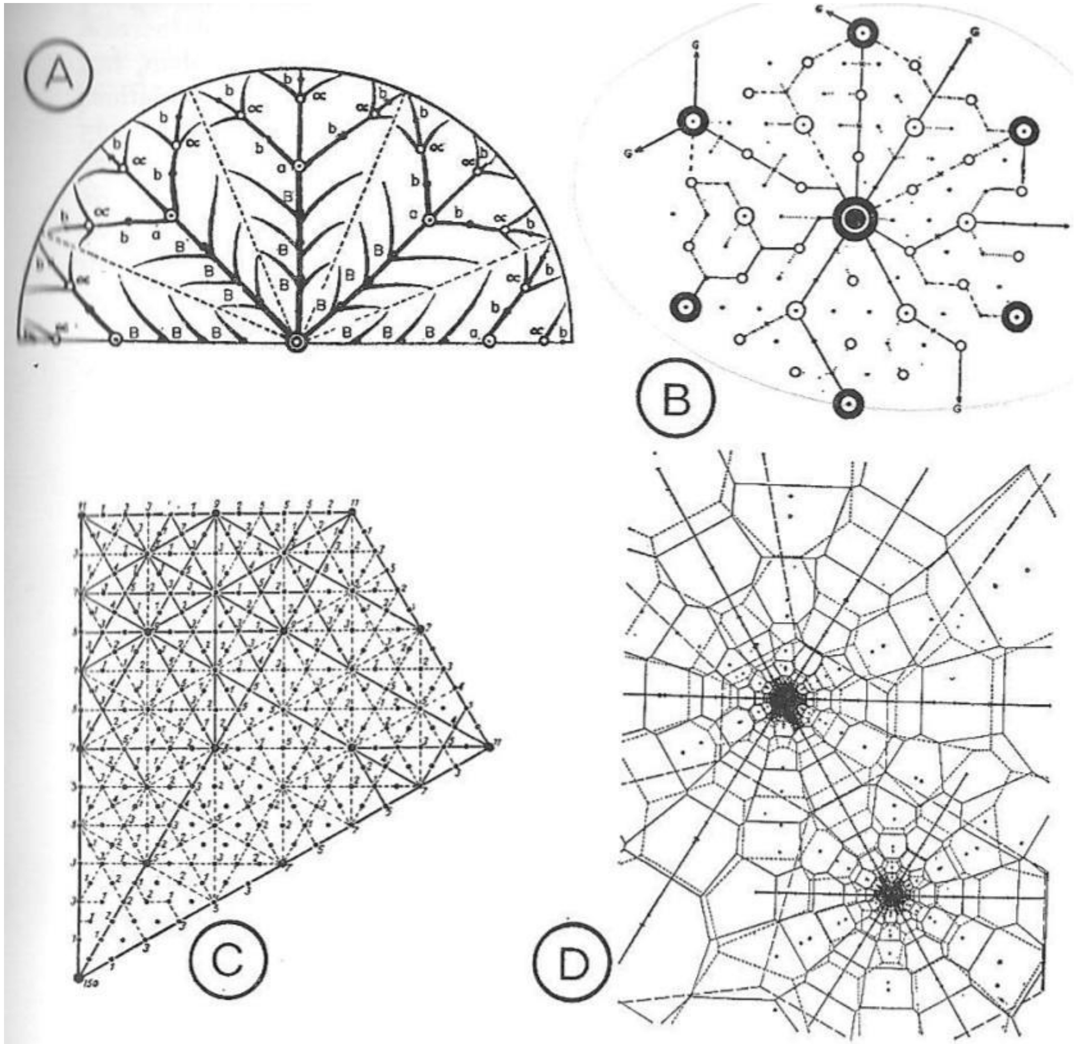


Fig. 3.11. Transport networks for theoretical settlement systems. (*A*) Kohl, 1850, (*B*) Christaller, 1933, (*C*) Lösch, 1954, (*D*) Isard, 1960. Source: Domanski, 1963, pp. 21–9.

[redacted]

Can we add anything to this?

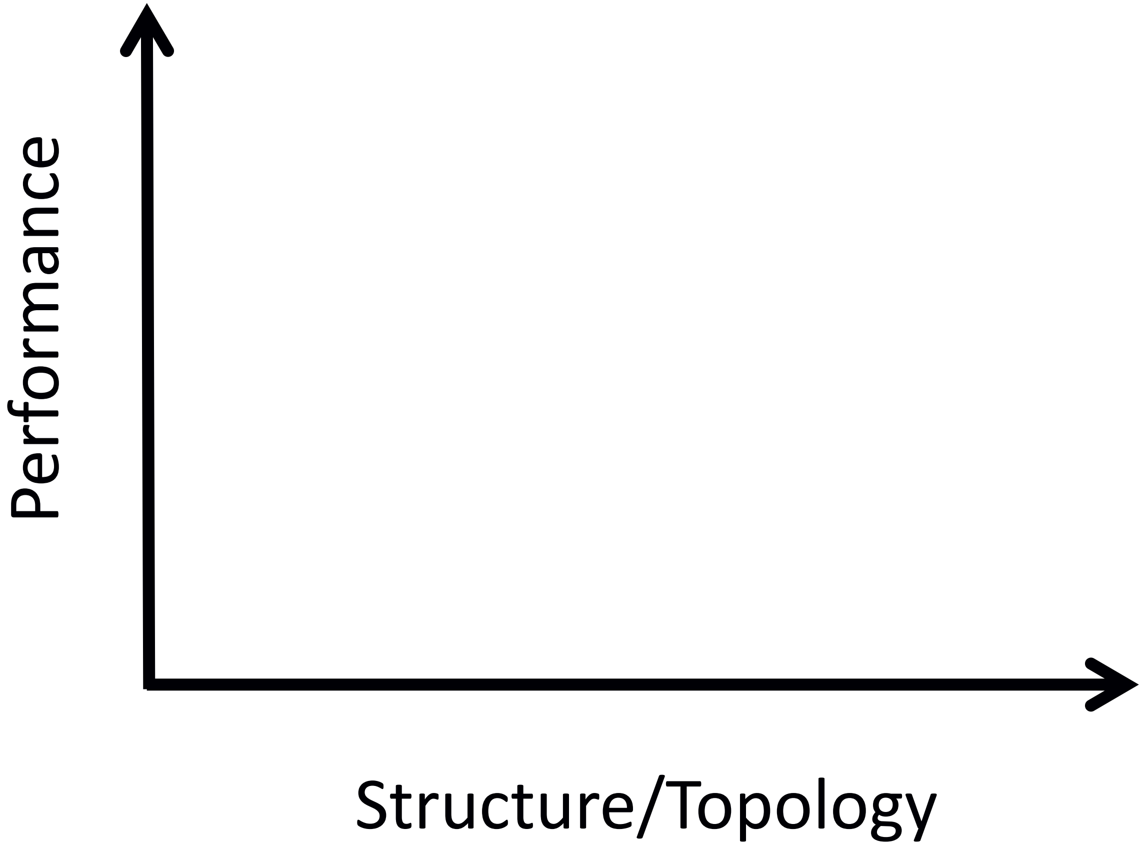
Who is interested in such questions?

Network Science

- In the last 20 years the research area of Network Science has exploded in terms of activity.
- Attempt to characterise the shape and structure of networks - often huge networks that are difficult to observe directly e.g. the WWW.
- Investigate how structure affects network properties (e.g. efficiency, vulnerability)

A simple idea to start with....

“Performance” vs “Structure”



How to proceed?

What is on the x-axis?
How to arrange networks in a meaningful order?

Metrics & measures from Network Science?

Typically non-planar no congestion no demand.

What is on the y-axis?

Price of Anarchy?
[Some work exists]

Depends on demand, which is another network. Tricky!

What methodology?

Use real networks?

Range?

Use synthetic networks?

How realistic?
Generation?

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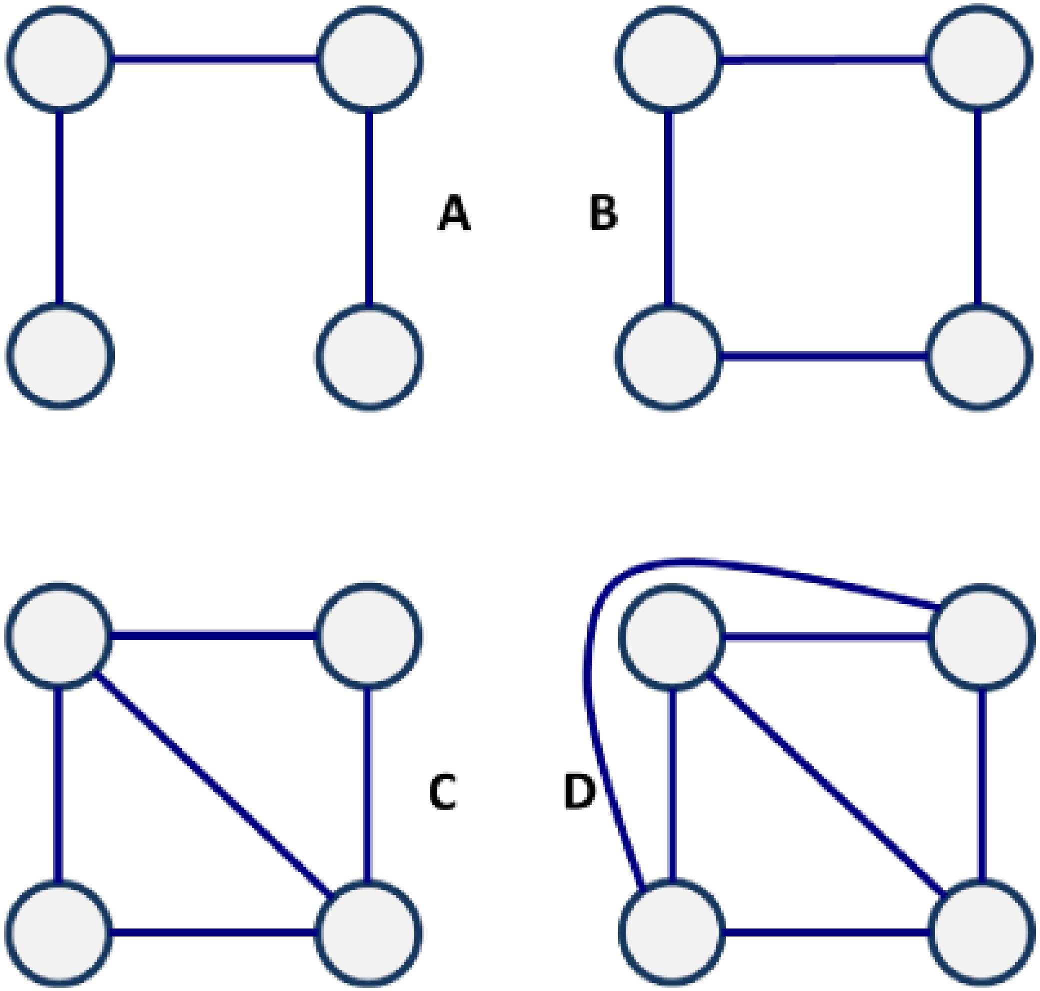
Example = Meshedness [= relative number of cycles]



$$M = \frac{m - n + 1}{2n - 5}$$

	Faces	2n-5	M
A	0	3	0.00
B	1	3	0.33
C	2	3	0.66
D	3	3	1.00

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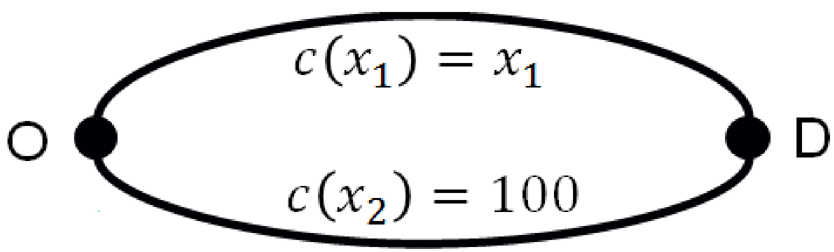
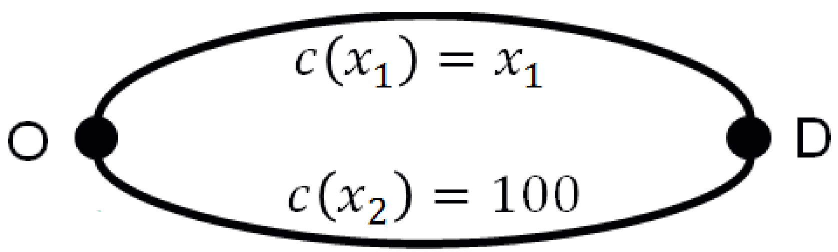
Routing in Road Networks

Selfish routing, to minimise individual travel cost = **User Equilibrium (UE)**.

This is known to be inefficient.

Total Network Travel Cost (TTC) is minimised by **System Optimal (SO)** routing





$q_{OD} = 100$

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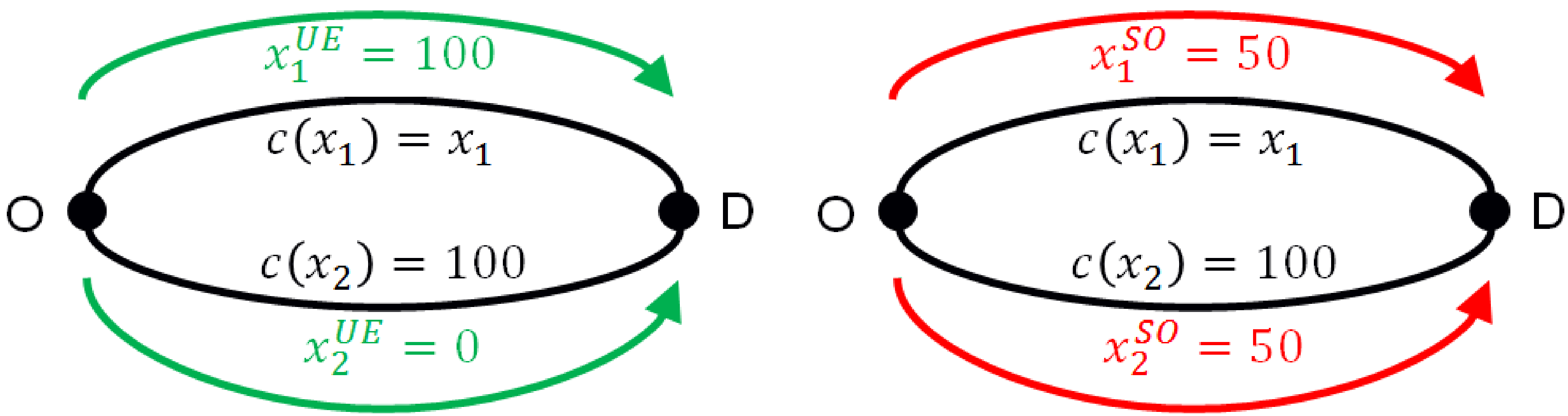

$q_{OD} = 100$ $\quad TTC^{UE} = 10{,}000$ $\quad TTC^{SO} = 7{,}500$

The Price of Anarchy

The ‘Price of Anarchy’ (PoA) is defined as the ratio of the total travel cost (TTC) under UE routing to the total travel cost under SO routing:

$$PoA = \frac{TTC_{UE}}{TTC_{SO}} = \frac{\sum_i x_i^{UE} c_i\left(x_i^{UE}\right)}{\sum_i x_i^{SO} c_i\left(x_i^{SO}\right)}$$

$1 \leq PoA$ and for linear link cost functions $PoA \leq \frac{4}{3}$

A (not tight) upper bound is given by:

$c_i = a_i + b_i x_i^{\beta_i}$ with all $\beta_i \leq p$

$$PoA \leq \left[1 - p/(p+1)^{(p+1)/p}\right]^{-1}$$

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Theoretical analysis? How?
Empirically test networks.

Use real networks?

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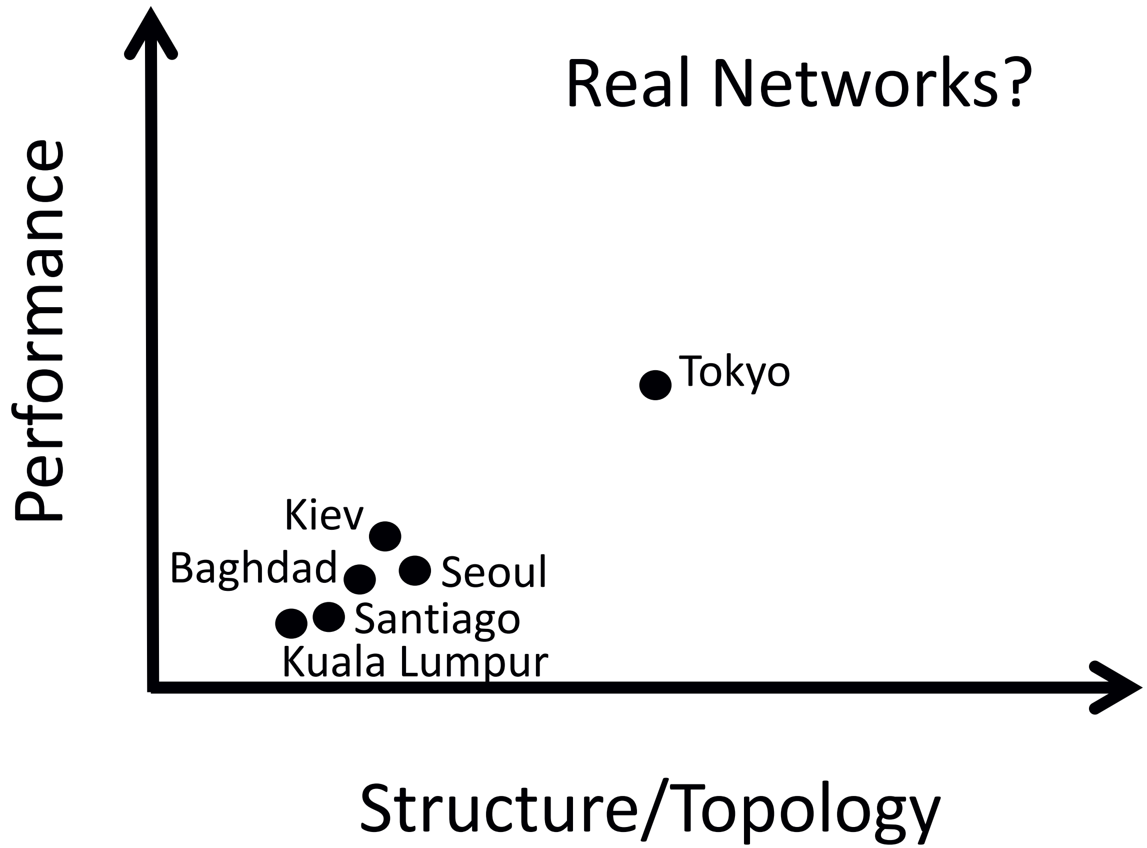
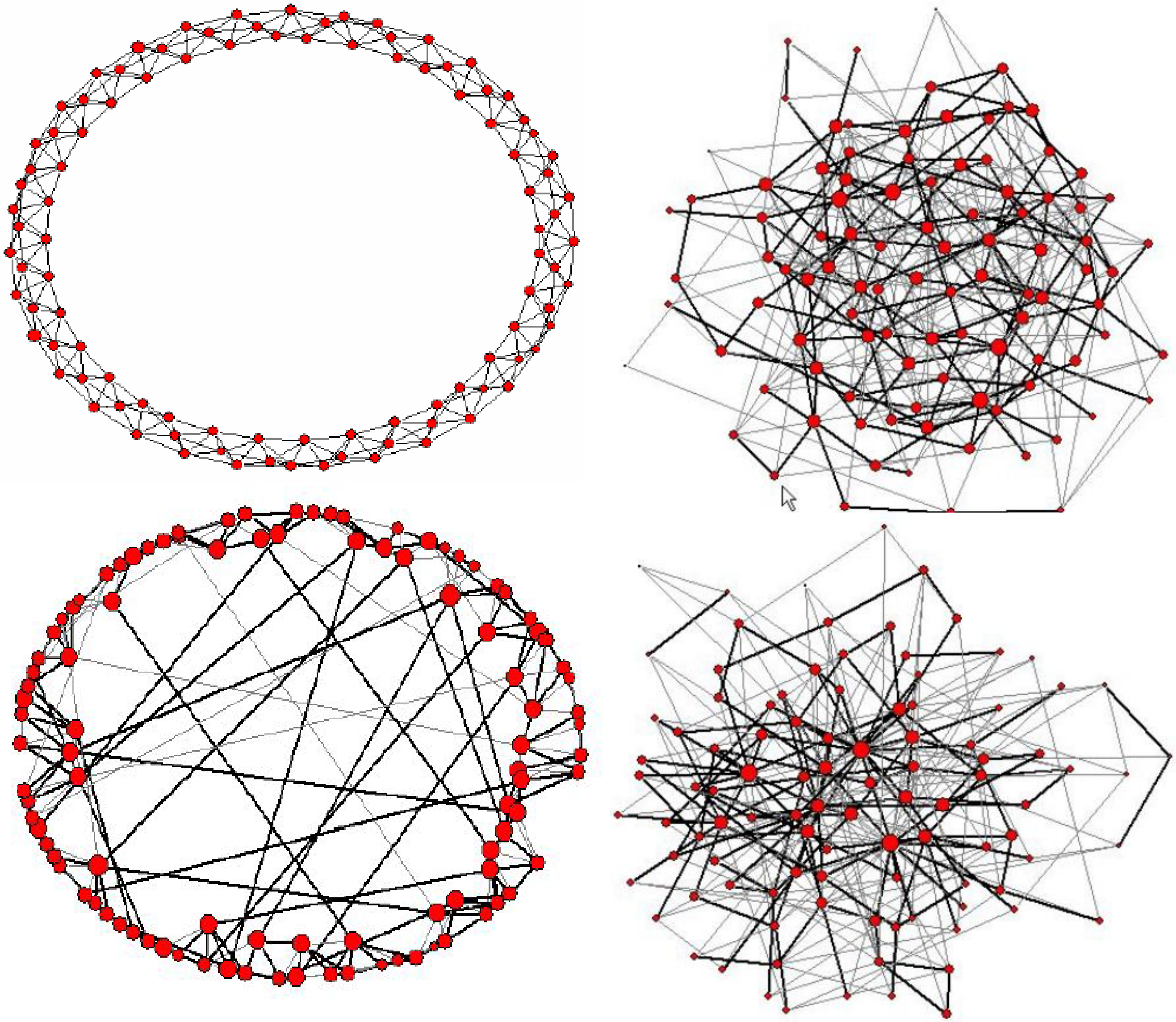
“Performance” vs “Structure”



Illustrative example from Network Science

Price of Anarchy & Network Topology



Link cost functions:

$$c_i = a_i + b_i x_i$$

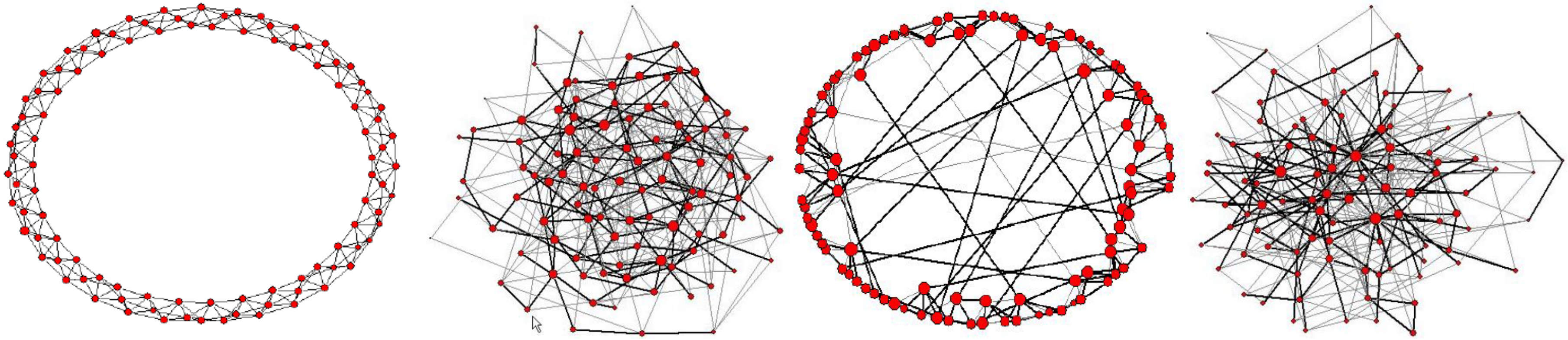
where a_i, b_i randomly selected from:

$$a_i = \{1,2,3\}$$

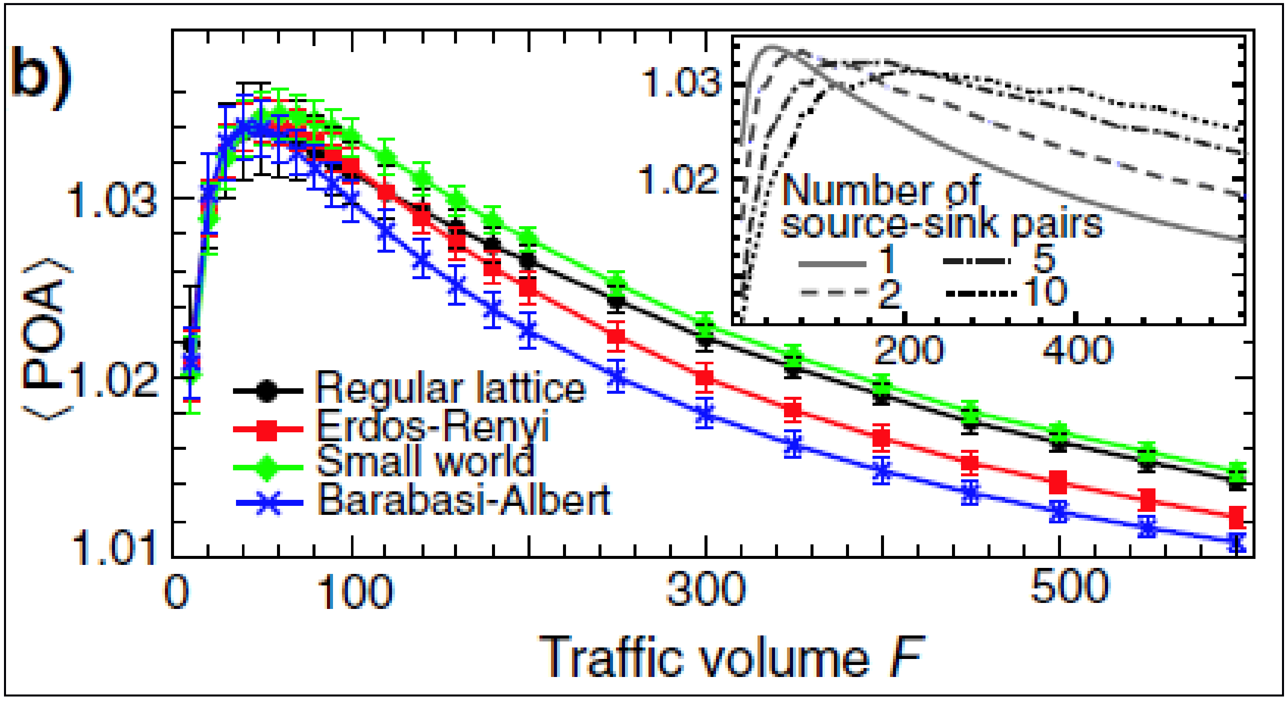
$$b_i = \{1,2, \dots, 100\}$$

Source: Youn et al. 2008. Physical Review Letters, 101.

Price of Anarchy & Network Topology



Regular Lattice | Erdos-Renyi (Random) | Small world | Barabasi-Albert (Scale-free)

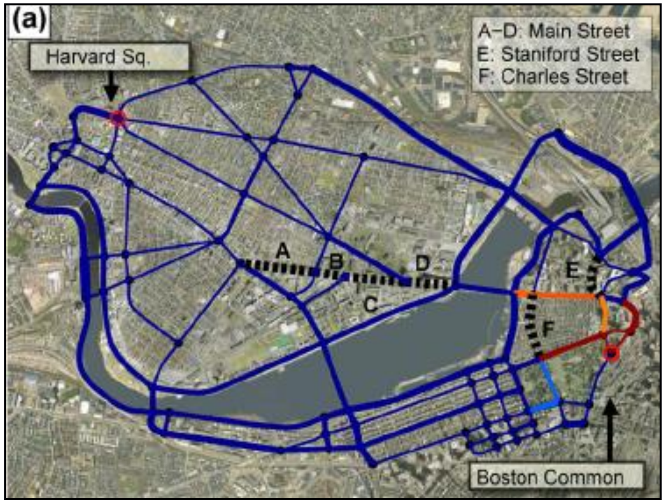


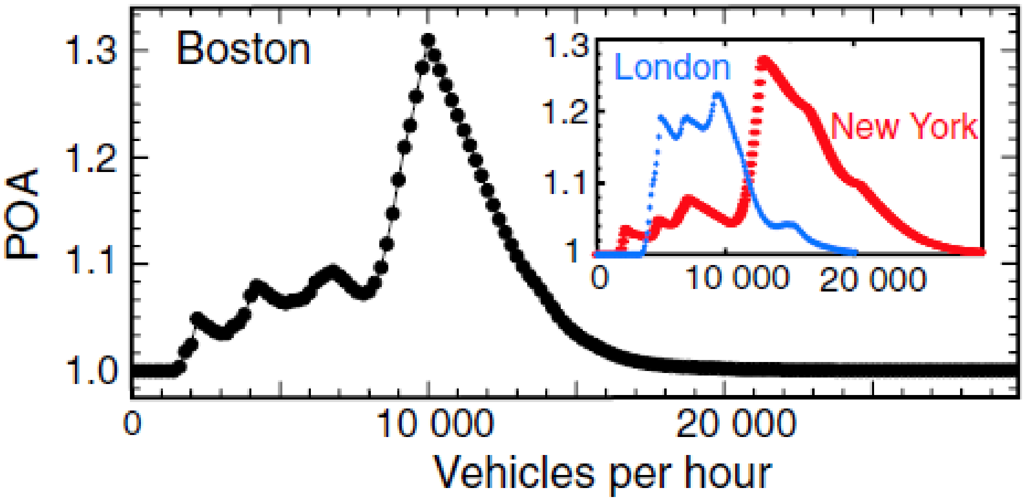
- Canonical Network Science Topologies
- Single OD pair [or 2,5,10 ODs]

Source: Youn et al. 2008. Physical Rev Letters 101.

PoA In Real Networks

Price of Anarchy does vary with demand and supply structure in real networks:





A single OD in each of 3 cities.

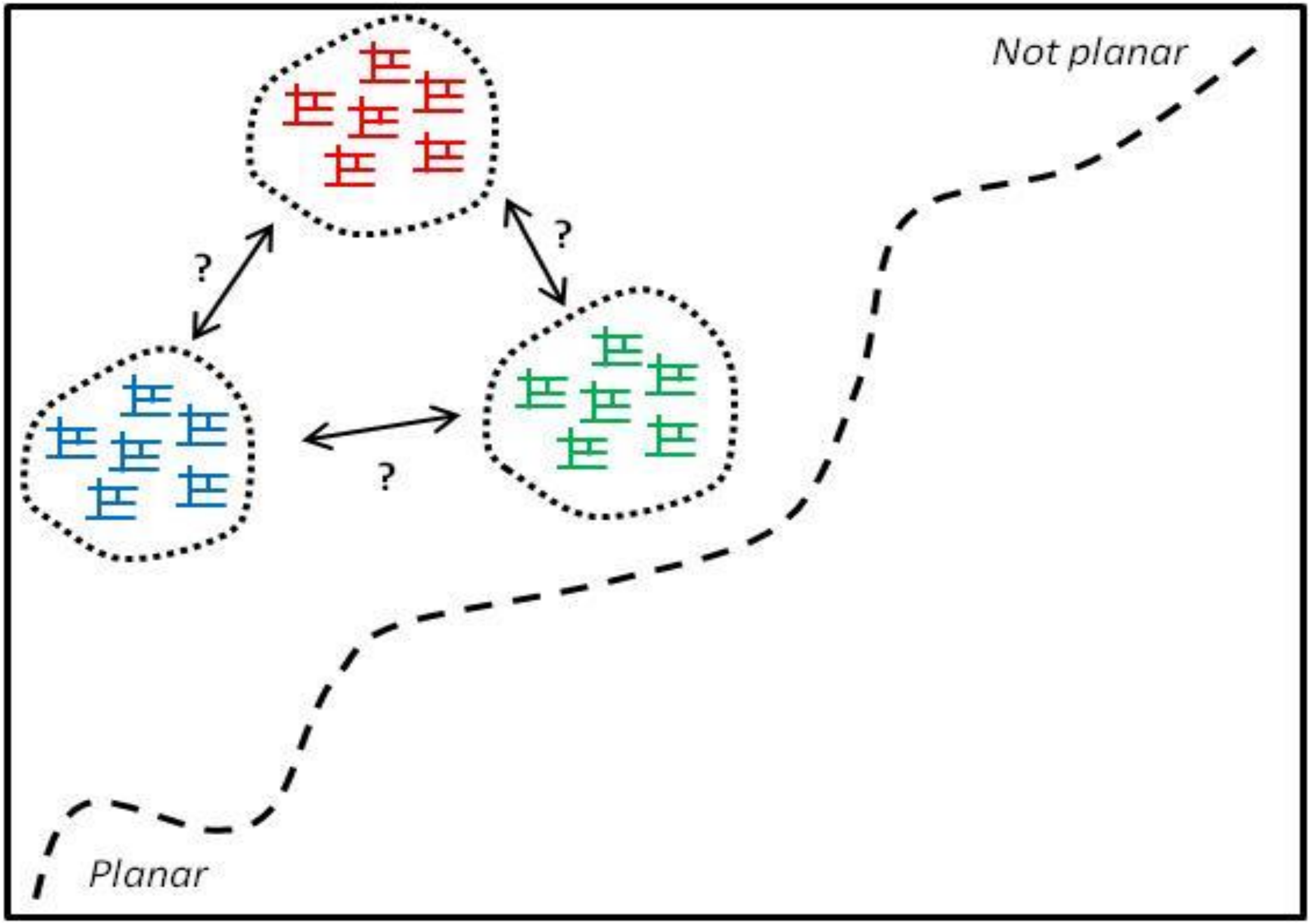
No explanation of what is going on.

Source: Youn et al. 2008. Phys Rev Letters, 101.

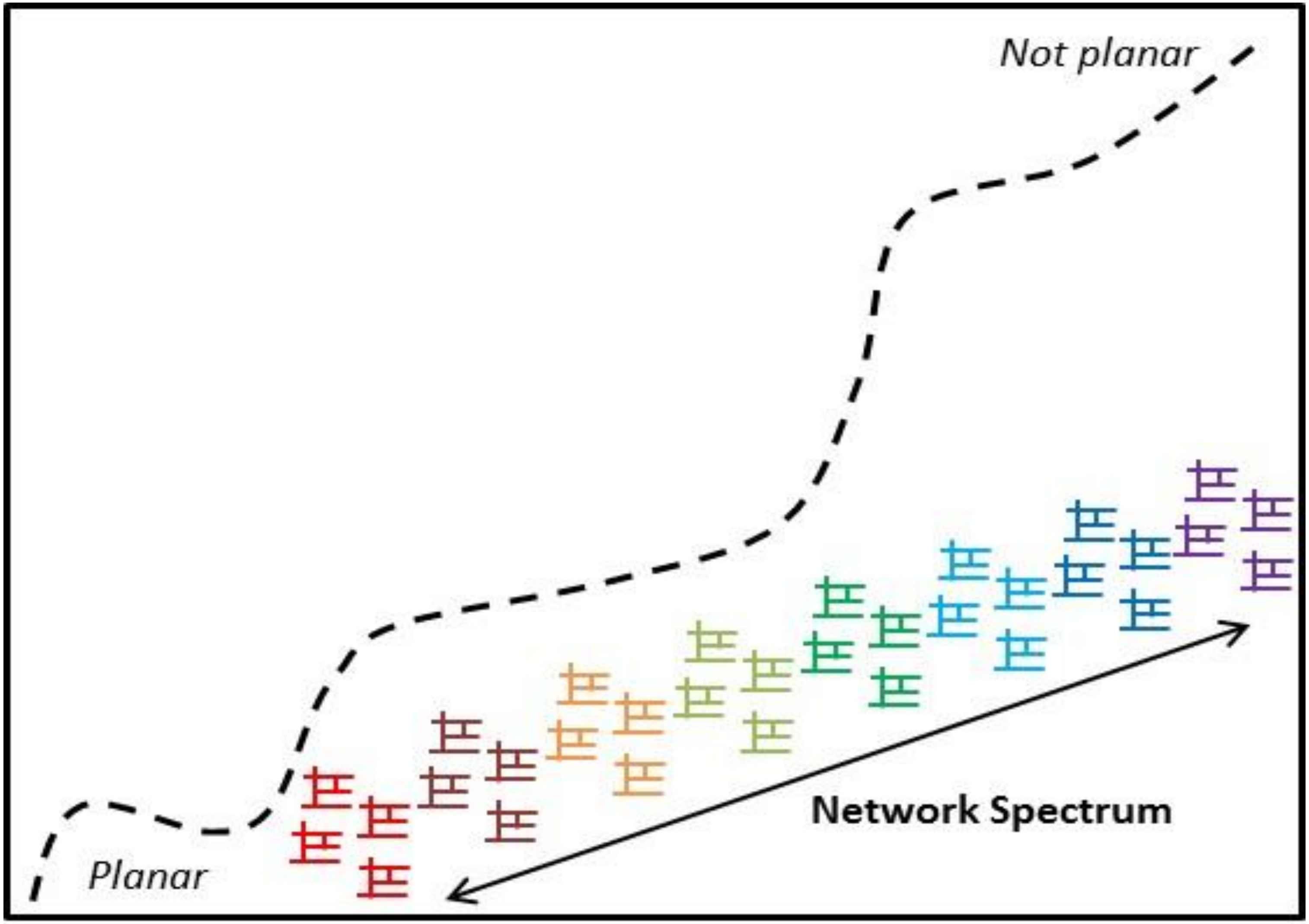
Structure vs Performance

Study	Topologies	Network Size ($\boldsymbol{n}$ nodes, $\boldsymbol{m}$ links, $\langle \boldsymbol{k} \rangle$: Average Node Degree)	Number of Network Realisations	Link Travel Time Functions $\boldsymbol{t_i}$ ($\boldsymbol{t_{0i}}$: free-flow travel time, $\boldsymbol{x_i}$: link flow, $\boldsymbol{cap_i}$: link capacity)	Demand Structure	Main Performance Indicator
Wu et al. (2006)	Random; Scale-Free; Small World	$\boldsymbol{n = 400}$; $\boldsymbol{m = 1400}$; $\langle \boldsymbol{k} \rangle = \boldsymbol{7}$	25	$\boldsymbol{t_i = t_{0i}\left[1 + 0.15\left(\frac{x_i}{cap_i}\right)^4\right]}$ - $\boldsymbol{t_{0i}} \in (\boldsymbol{0, 0.1}]$ randomly selected for each link - $\boldsymbol{cap_i} \in [\boldsymbol{20, 60}]$ randomly selected for each link	Random	Proportion of links over Capacity
Zhao and Gao (2007)	Regular Ring; Random; Scale-Free; Small World	$\boldsymbol{n = 500}$; $\boldsymbol{m = 1000}$; $\langle \boldsymbol{k} \rangle = \boldsymbol{4}$	50	$\boldsymbol{t_i = t_{0i}\left[1 + 0.15\left(\frac{x_i}{cap_i}\right)^4\right]}$ - $\boldsymbol{t_{0i}} \in (\boldsymbol{0, 1}]$ randomly selected for each link - $\boldsymbol{cap_i = 10000}$ for each link	Uniform	Total Travel Time
Youn et al. (2008)	1D Regular Lattice; Random; Scale-Free; Small World	$\boldsymbol{n = 100}$; $\boldsymbol{m = 300}$; $\langle \boldsymbol{k} \rangle = \boldsymbol{6}$	50	$\boldsymbol{t_i = a_i + b_i x_i}$ - $\boldsymbol{a_i} \in \{\boldsymbol{1, 2, 3}\}$ randomly allocated to each link - $\boldsymbol{b_i} \in \{\boldsymbol{1, 2, \ldots, 100}\}$ randomly allocated to each link	Single OD pair	Price of Anarchy
Wu et al. (2008a)	Random; Scale-Free	$\boldsymbol{n = 100}$; $\boldsymbol{m = 1350}$; $\langle \boldsymbol{k} \rangle = \boldsymbol{2.7}$	100	$\boldsymbol{t_i = t_{0i}\left[1 + 0.15\left(\frac{x_i}{cap_i}\right)^4\right]}$ - $\boldsymbol{t_{0i}} \in (\boldsymbol{0, 0.1}]$ randomly selected for each link - $\boldsymbol{cap_i = C}\ \forall \boldsymbol{i}$ but the value of $\boldsymbol{C}$ is not defined	Not defined	Proportion of links over capacity
Wu et al. (2008b)	Regular Lattice; Random; Scale-Free; Small World	$\boldsymbol{n = 100, \ldots, 1000}$; $\boldsymbol{m = 100, \ldots, 1000}$; $\langle \boldsymbol{k} \rangle = \boldsymbol{2}$	50	$\boldsymbol{t_i = t_{0i}\left[1 + 0.15\left(\frac{x_i}{cap_i}\right)^4\right]}$ - $\boldsymbol{t_{0i}} \in (\boldsymbol{0, 1}]$ randomly selected for each link - $\boldsymbol{cap_i}$ is not defined	Random	Price of Anarchy
Sun et al. (2012)	Scale-Free with variable community structure	$\boldsymbol{n = 100, 160, 220}$; $\boldsymbol{m = 400, 640, 880}$; 4 communities	20	$\boldsymbol{t_i = t_{0i}\left[1 + 0.15\left(\frac{x_i}{cap_i}\right)^4\right]}$ - $\boldsymbol{t_{0i}}$ randomly selected for each link - $\boldsymbol{cap_i = 60}\ \forall \boldsymbol{i}$	Random	Proportion of links over capacity
Zhu et al. (2014)	Scale-Free; Small World	$\boldsymbol{n = 1000}$; $\boldsymbol{m = 3000}$; $\langle \boldsymbol{k} \rangle = \boldsymbol{6}$	Not defined	$\boldsymbol{t_i = t_{0i}\left[1 + 0.15\left(\frac{x_i}{Ce_i}\right)^4\right]}$ - $\boldsymbol{t_{0i} = 1}$ for every link $\boldsymbol{i}$ between nodes $\boldsymbol{i1}$ and $\boldsymbol{i2}$ - $\boldsymbol{Ce_i = \min(Cn_{i1}/k_{i1}, Cn_{i2}/k_{i2})}$, for which i) $\boldsymbol{Cn_j}$ is fixed and ii) $\boldsymbol{Cn_j = f(k_j)}$	Uniform; Gravity Model	Volume to Capacity ratio (V/C)

Network Science Examples



What we want...a spectrum of networks



How to proceed?

What is on the x-axis?
How to arrange networks in a meaningful order?

Metrics & measures from Network Science?

Typically non-planar
no congestion
no demand.

What is on the y-axis?

Price of Anarchy?
[Some work exists]

Depends on demand, which is another network. Tricky!

What Methodology?
Theoretical analysis? How?
Empirically test networks.

Use real networks?

Use synthetic networks?

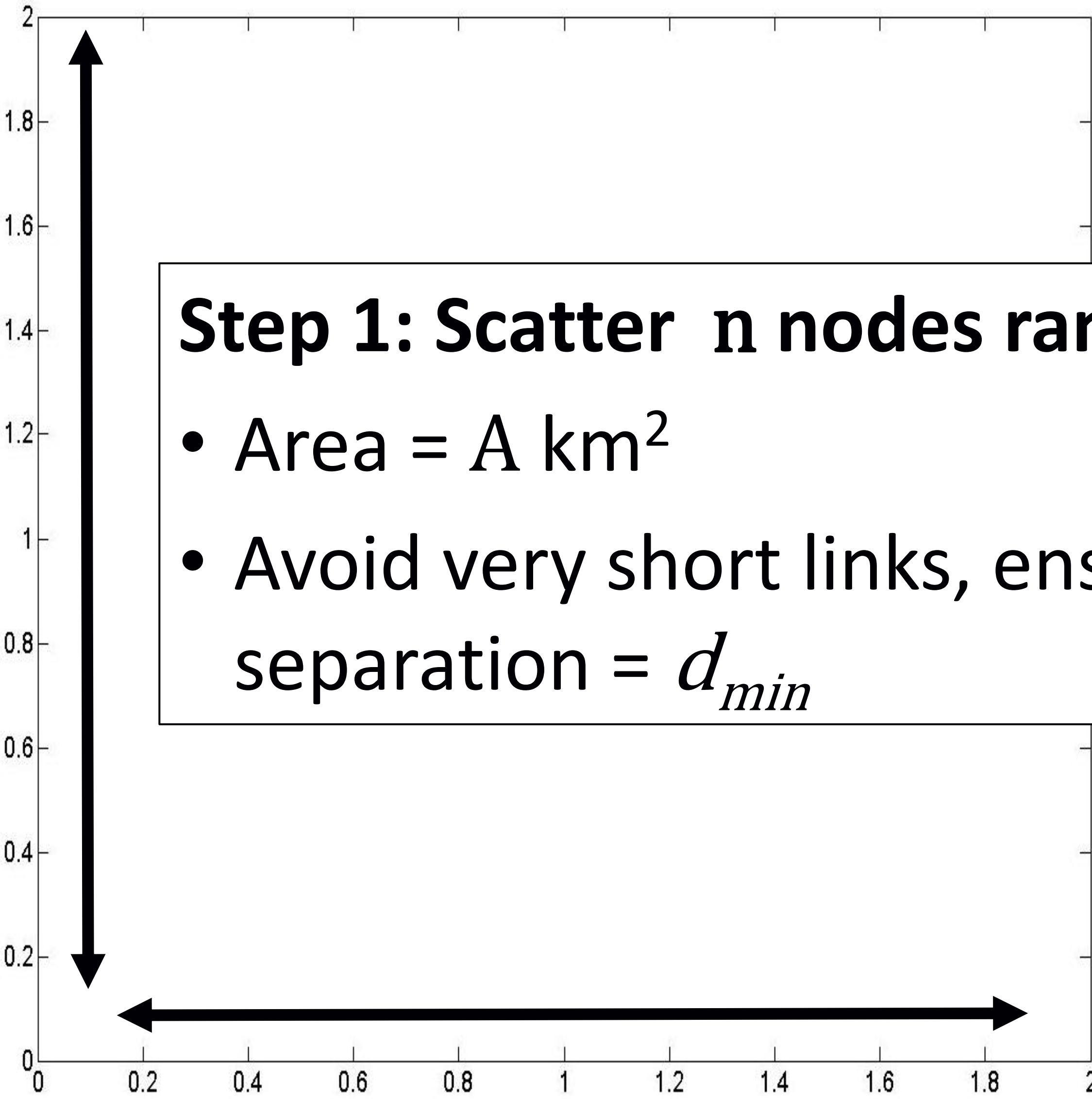
Range?

How realistic?
Generation?

What did we do?

1. We examined performance (PoA) across a “spectrum” of synthetic road-like networks.
2. We tried to understand the “mountain peaks” structure of PoA as a function of demand.

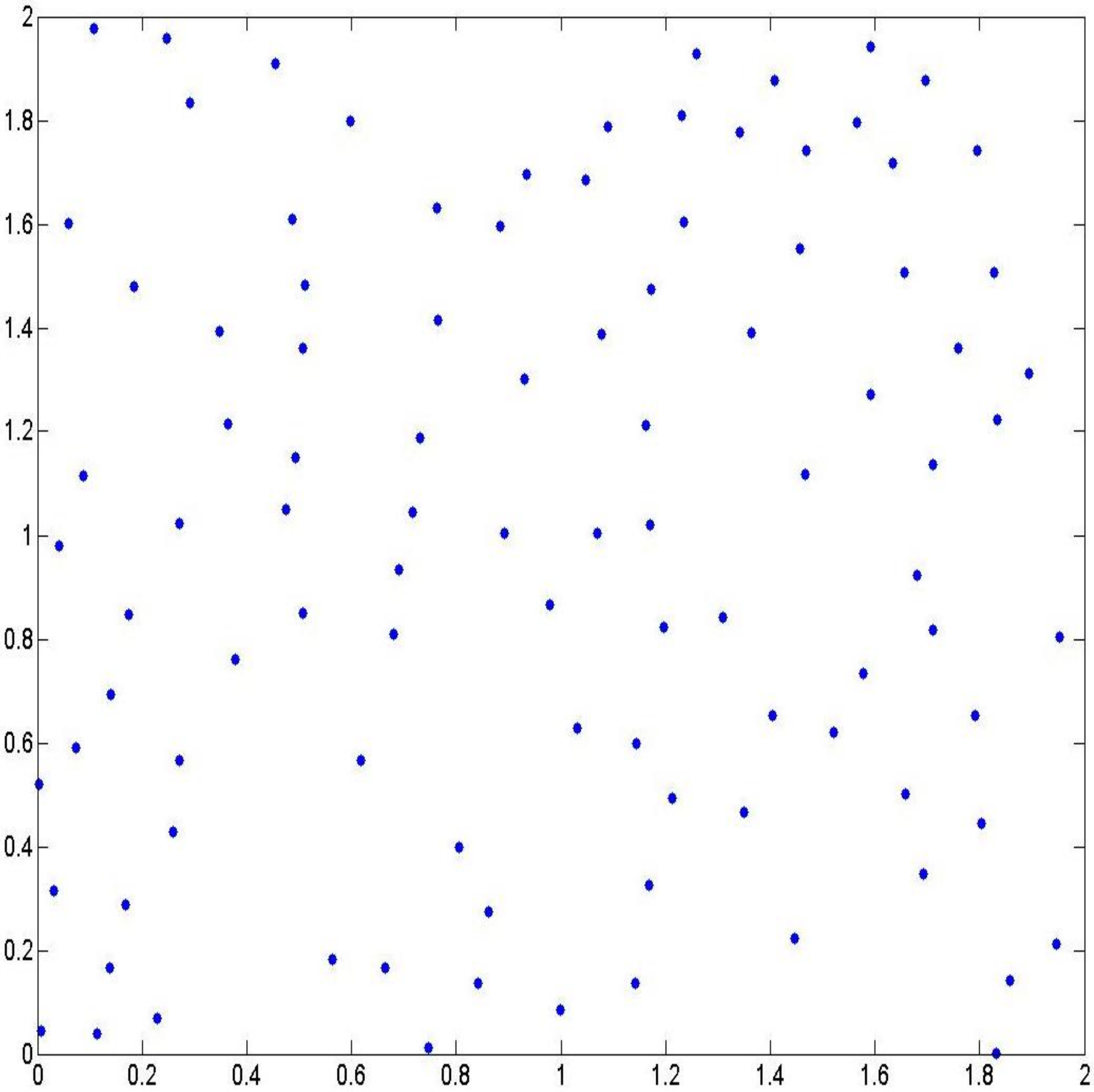
The Generation of a Spectrum of Road Network Topologies – Node Generation



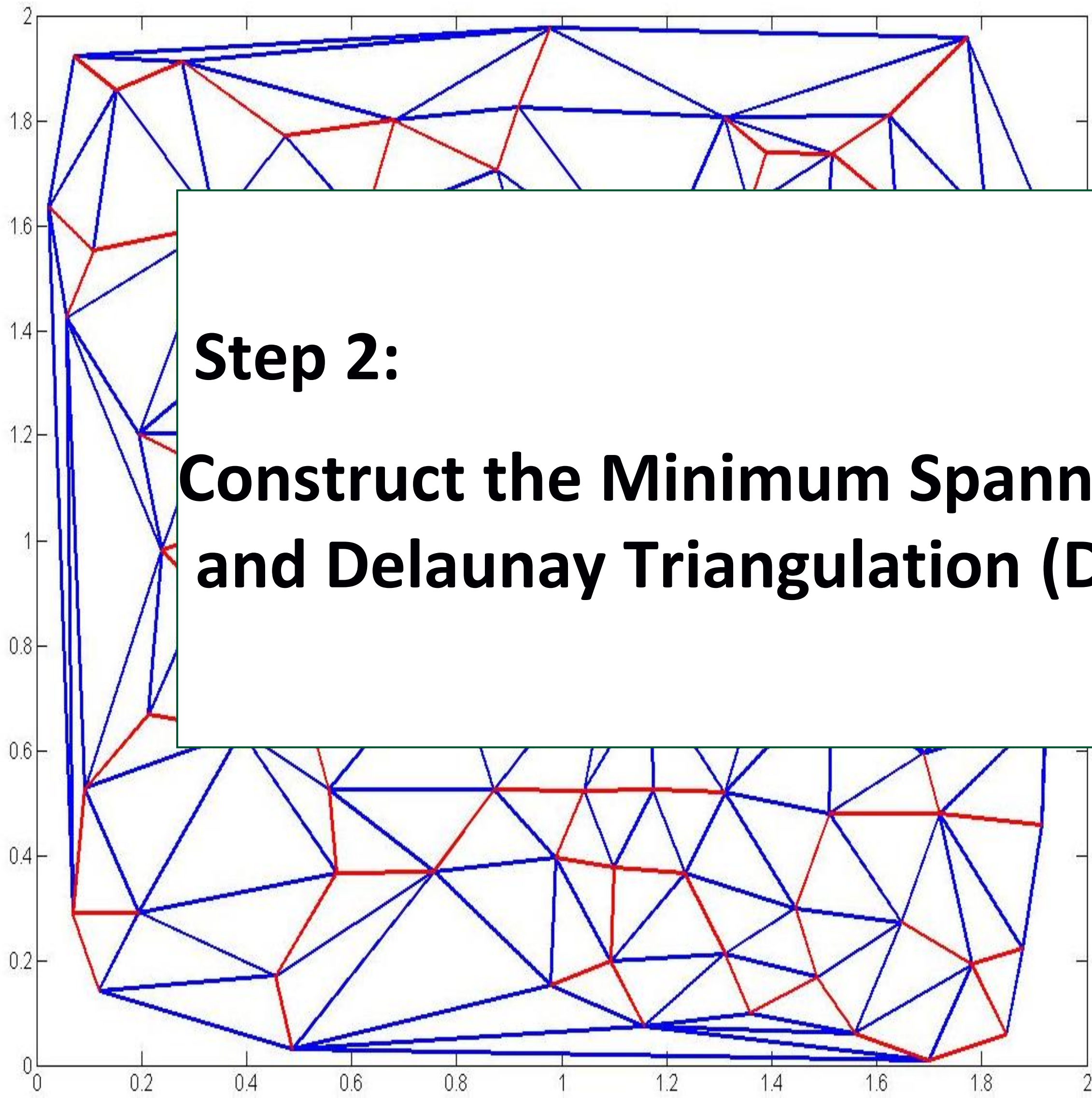
Step 1: Scatter n nodes randomly

- Area = A km^2
- Avoid very short links, ensure minimum node separation = d_{min}

The Generation of a Spectrum of Road Network Topologies – Node Generation



Delaunay Triangulation



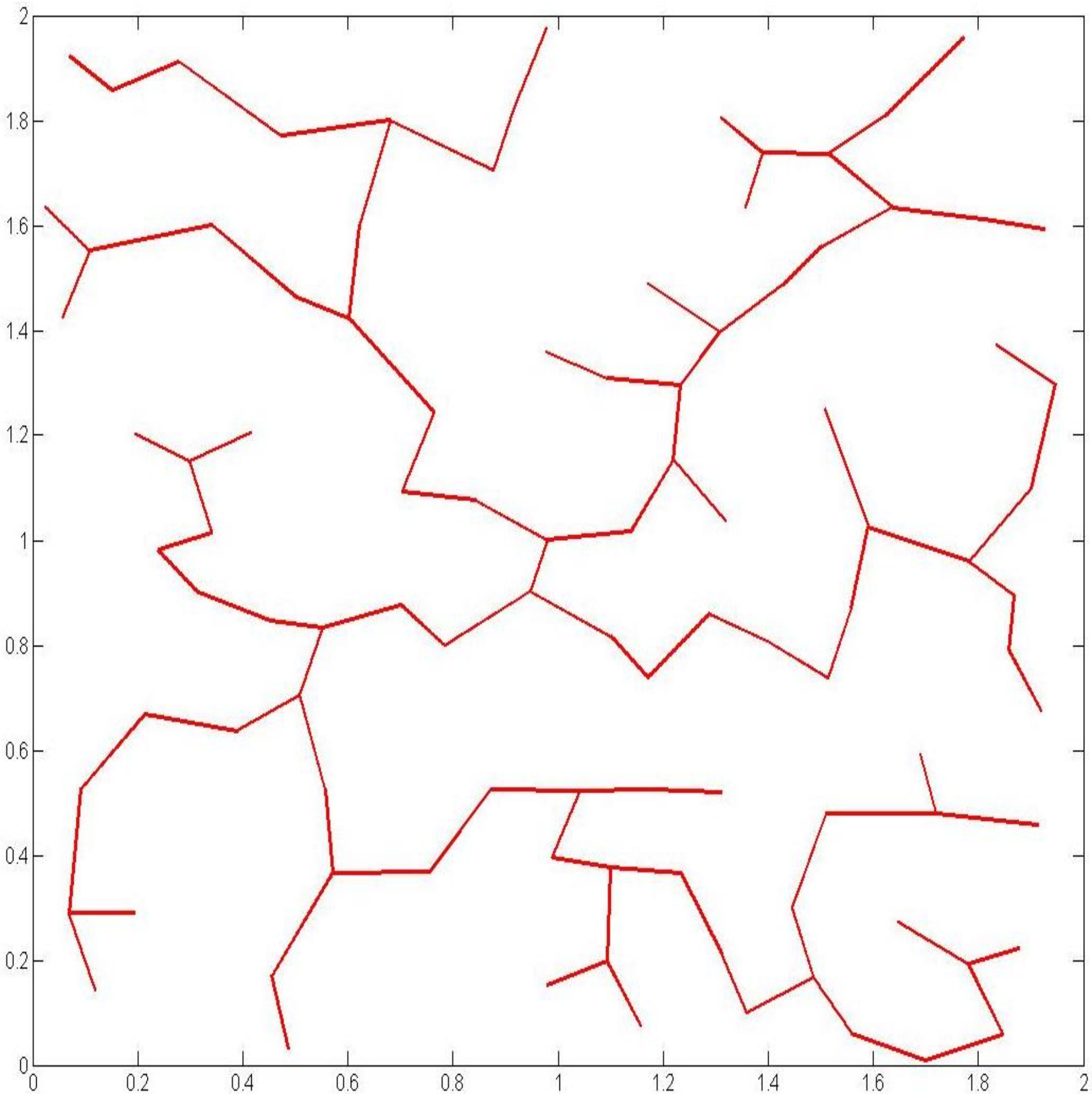
$$\frac{m}{\ } = \frac{284}{\ } = 2.84$$

Step 2:

Construct the Minimum Spanning Tree (MST) and Delaunay Triangulation (DT)

$$M = \frac{m-n+1}{2n-5} = 0.948$$

Minimum Spanning Tree



$$\frac{m}{n} = \frac{99}{100} = 0.99$$

$$M = \frac{m - n + 1}{2n - 5} = 0$$

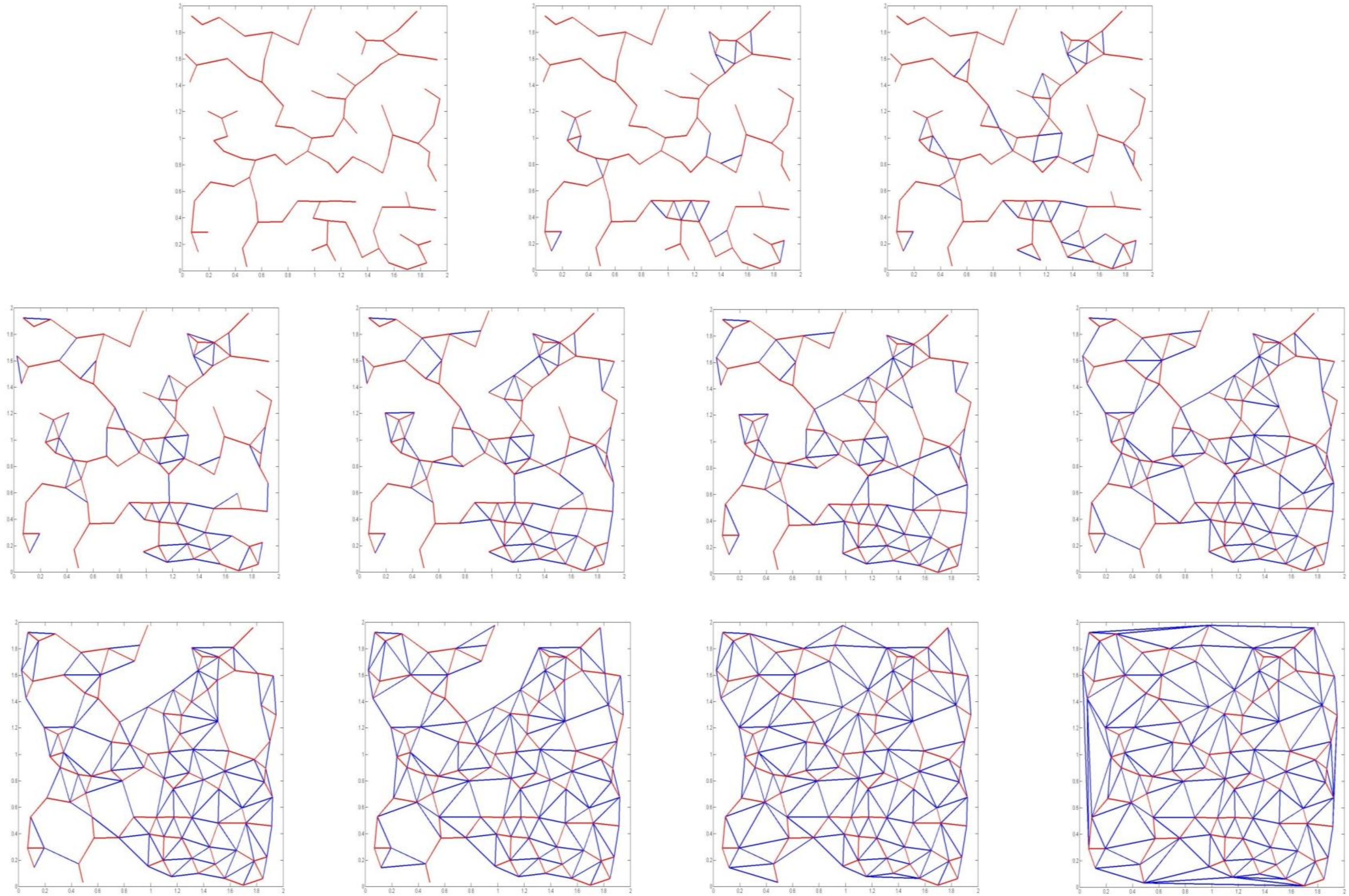
Aside: MST is always a subgraph of the DT

The Generation of a Spectrum of Road Network Topologies – Generation of the Spectrum

Step 3: Construct spectrum

Start with MST and add links from the DT (add links randomly to get total links = m)

Spectrum of Networks



[redacted]

Link cost functions

$$t_i = \frac{d_i}{48}\left[1 + 0.15\left(\frac{x_i}{800}\right)^4\right]$$

OD demand = q_r between each pair of nodes

Define demand density ρ_{dem}

$$q_r = \frac{A\rho_{dem}}{n(n-1)}$$

[redacted]

- Not totally "realistic" as road networks
- More plausible than most (all?) existing studies of this sort
- Controllable. We can generate a spectrum as desired
- = first try. Lots of ways to improve details but general experimental method is the key.

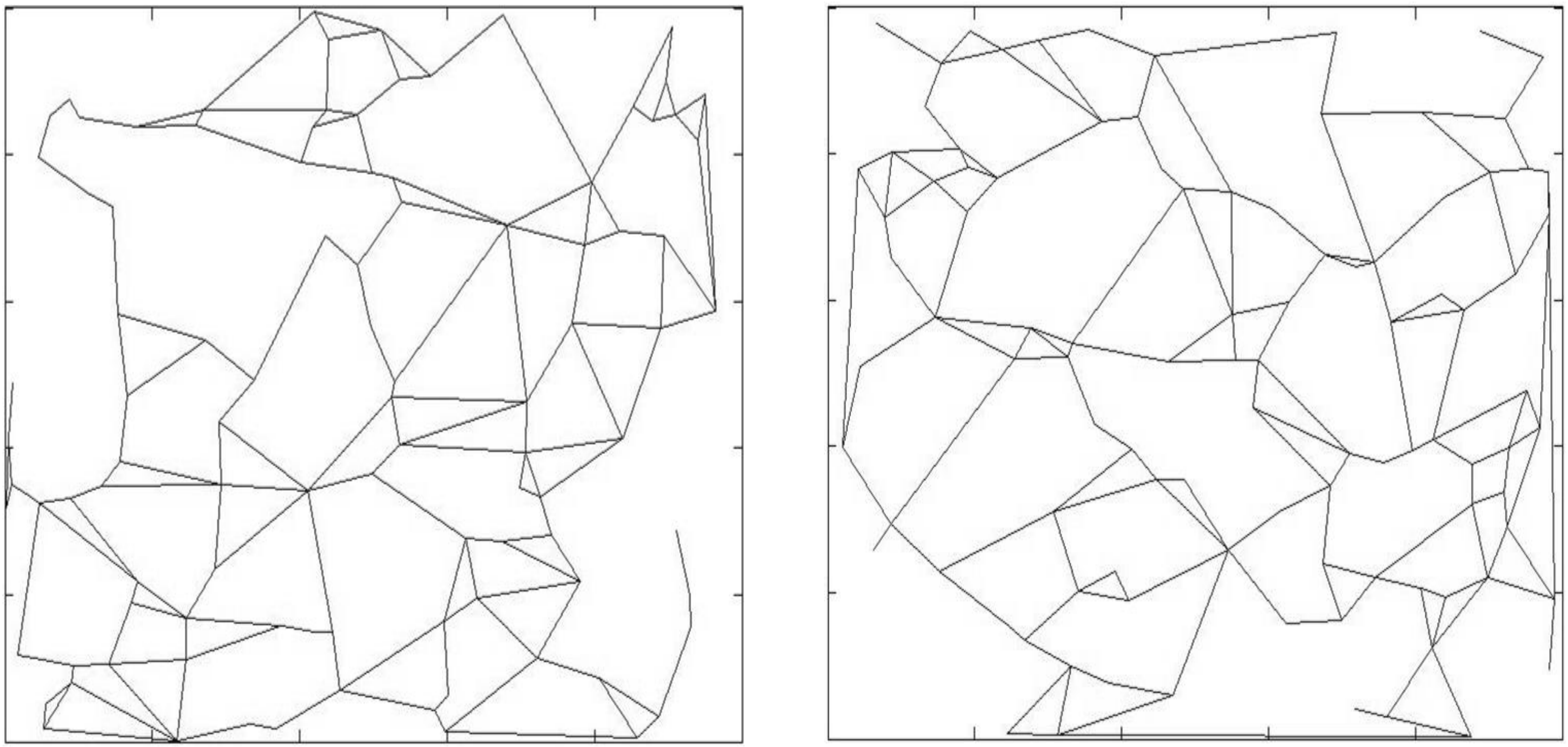
(admission: not quite first try!!)

Experiments

Experiment	Domain Size	N. Nodes	Node Density	N. Links	Meshdness	Demand Density
1. Demand Density	6.25	100	16	158	0.3	1250, 1300, …, 7900, 7950
2. Network Size	1.25, 1.875, …, 30.625, 31.25	20, 30, …, 490, 500	16	30, 46, …, 782, 798	0.3	4350
3. Network Density	6.25	20, 25, …, 295, 300	3.2, 4, …, 47.2, 48	30, 38, …, 470, 478	0.3	4350
4. Network Connectivity	6.25	100	16	99, 104, …, 284, 289	0, 0.03, …, 0.95, 0.97	4350

= Total of 28,000 Networks solved for both UE and SO

Demand Density

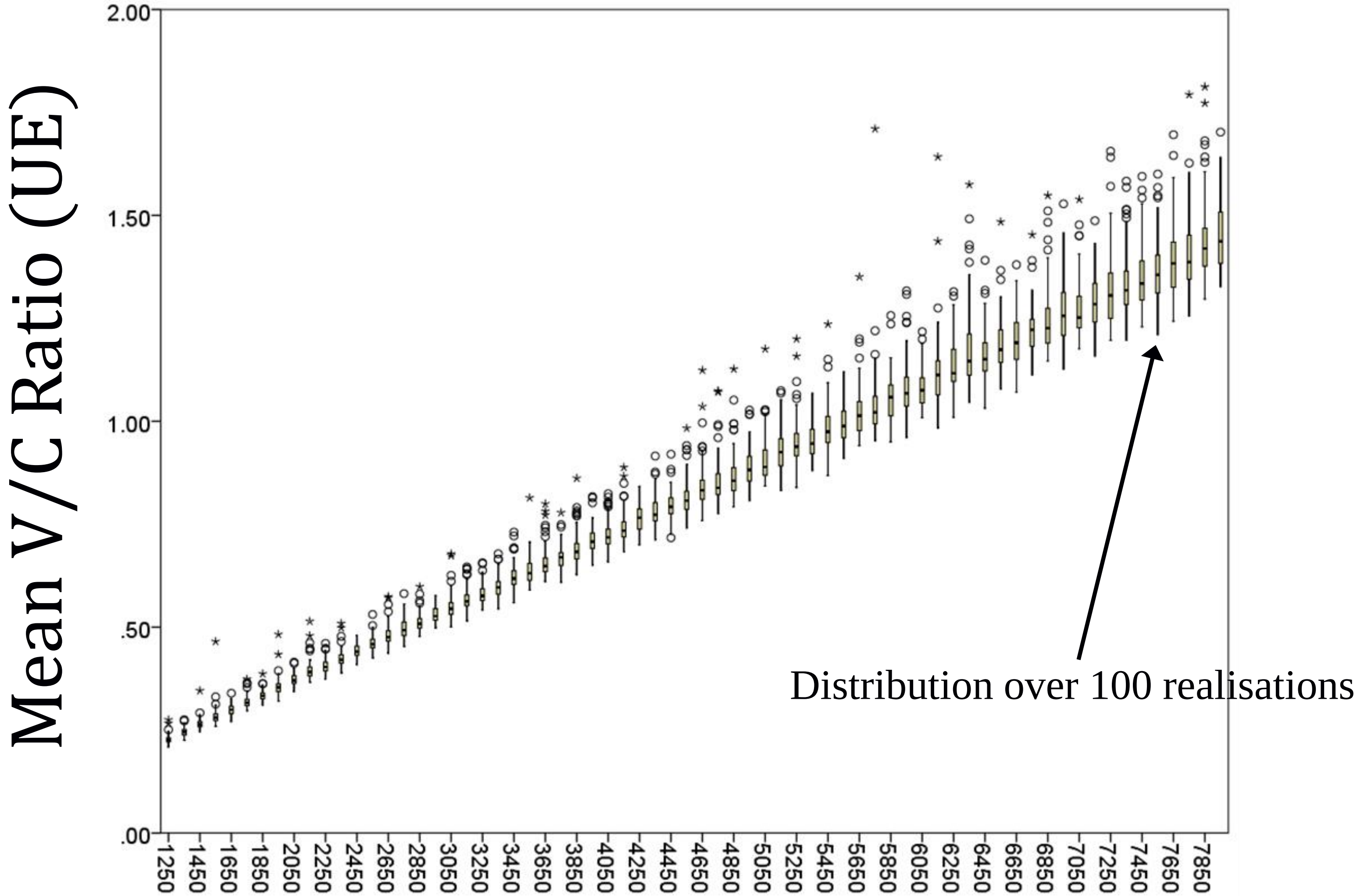


$n = 100$, $A = 6.25$, $d_{min} = 0.05$, ρ_n=16

meshedness = 0.3

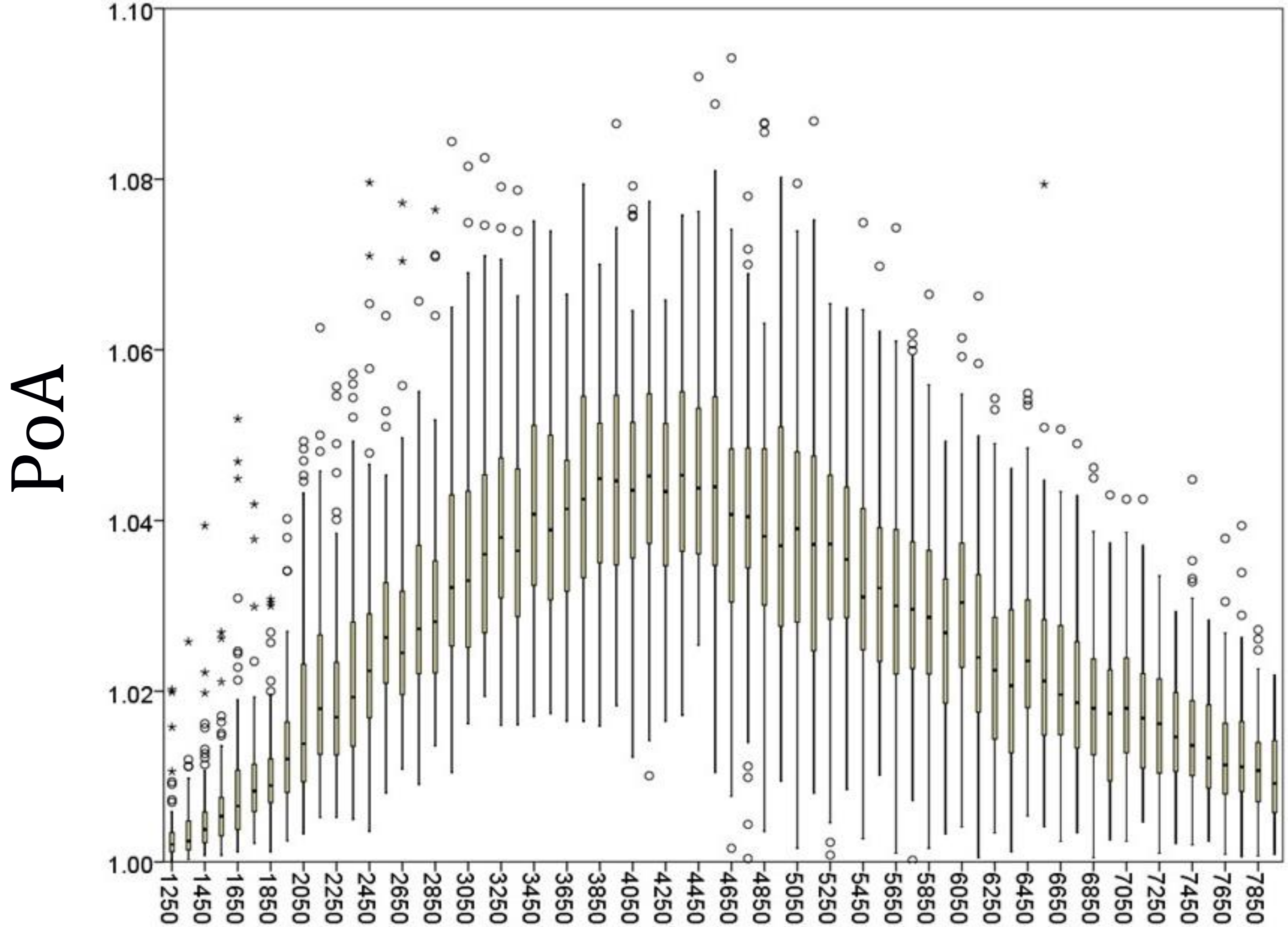
100 networks generated for each parameter setting

Demand Density



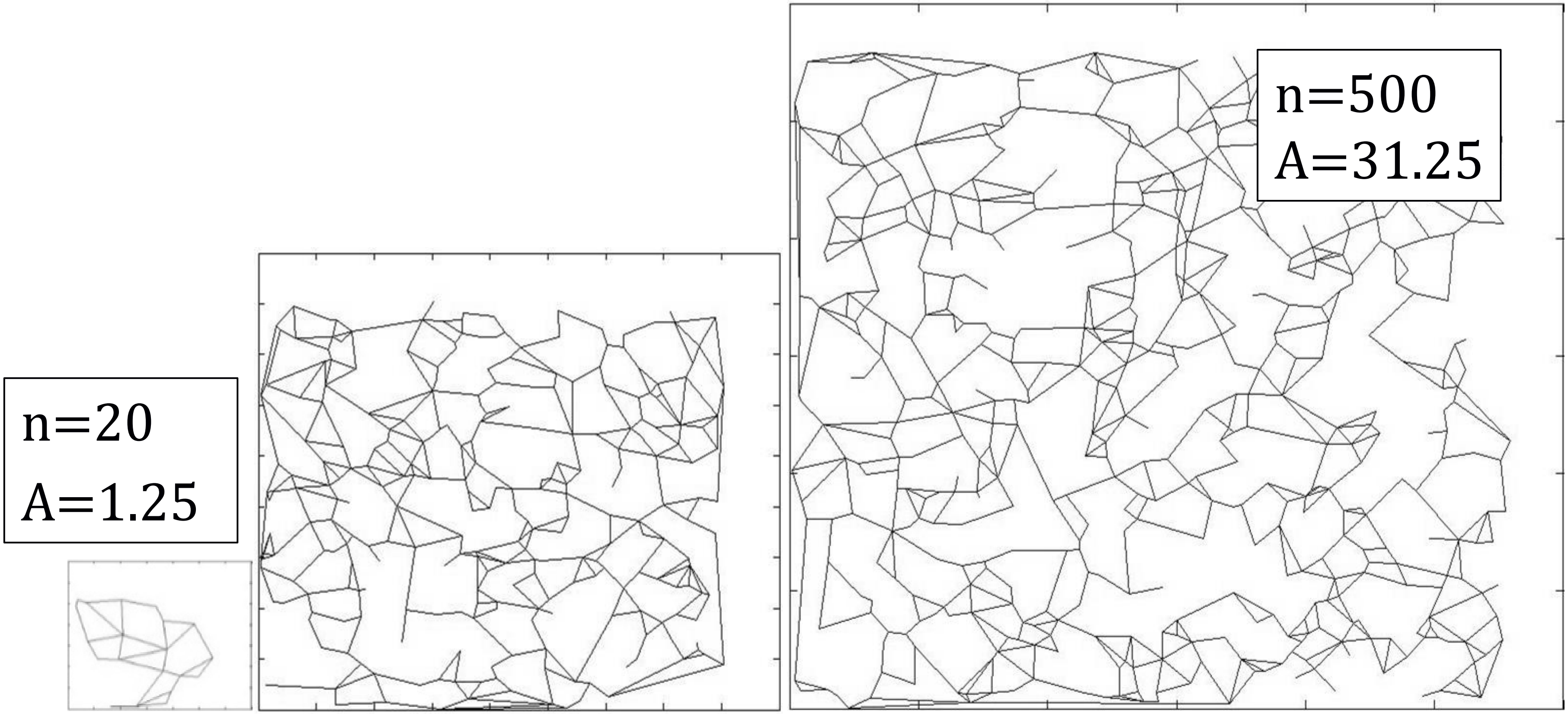
$\rho_{dem} \in [1250,7950]$

Demand Density



$\rho_{dem} \in [1250,7950]$

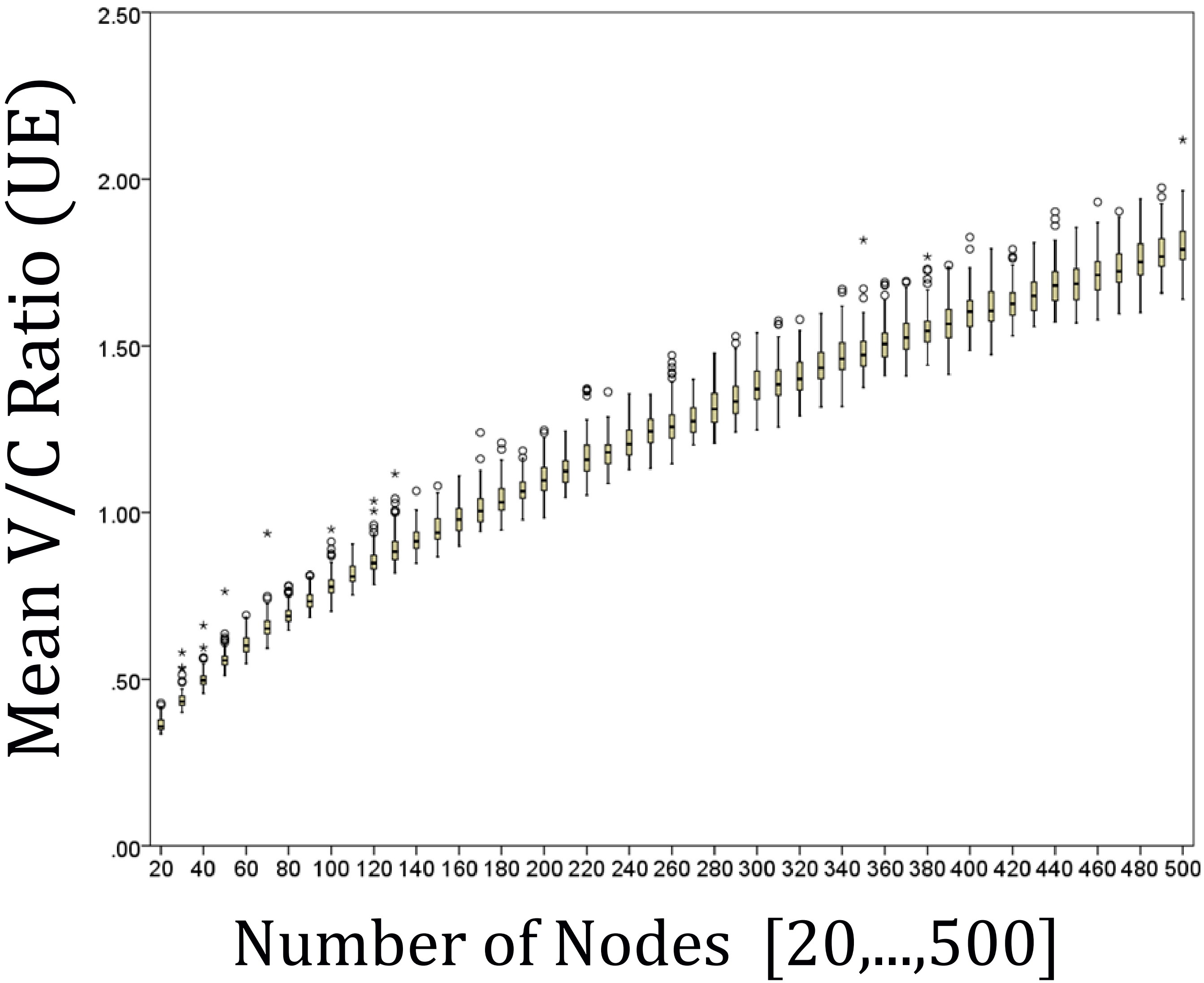
Network Size [& number of nodes & links increasing]



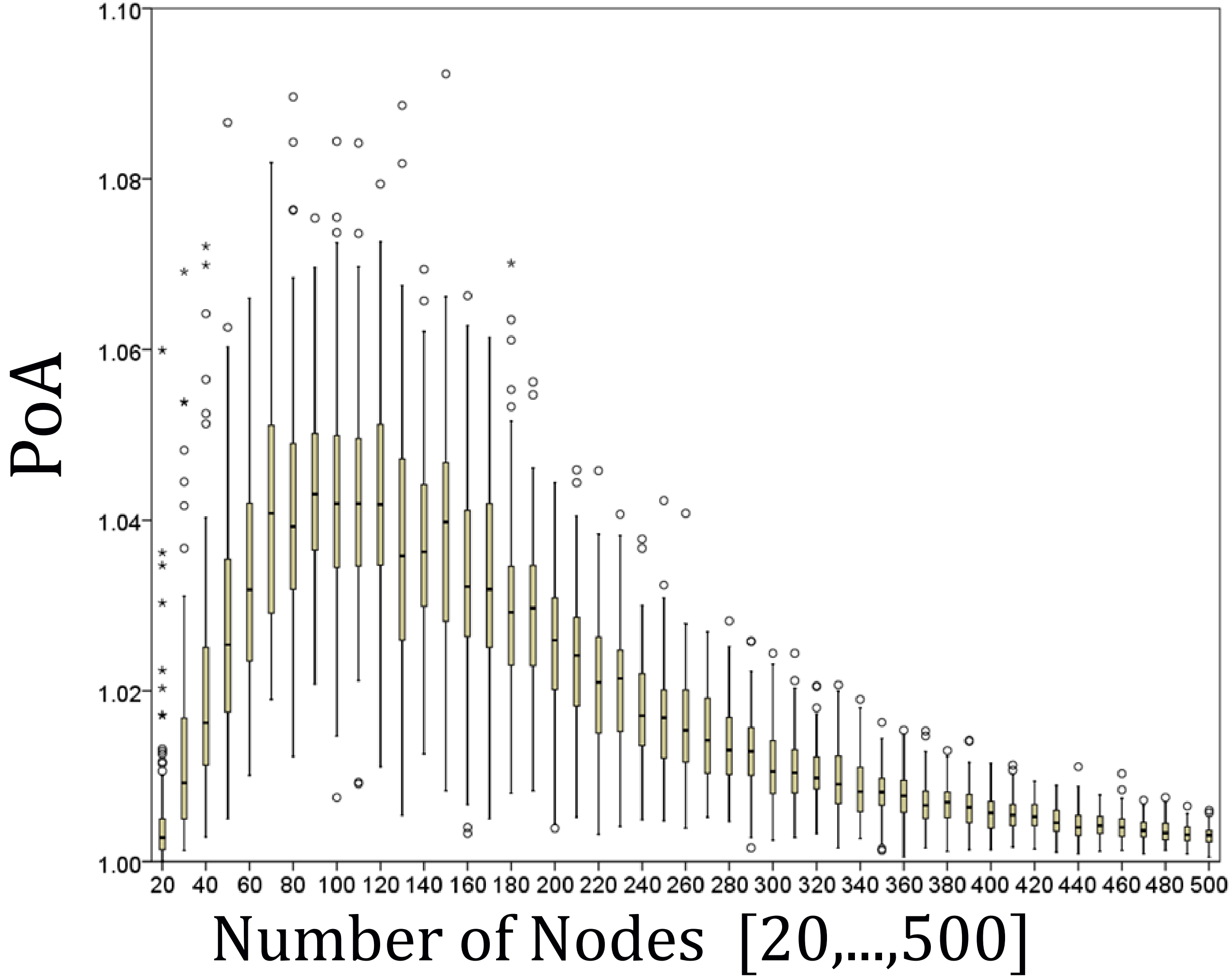
d_{min} = 0.05, ρ_n =16 $\rho_{dem} = 4350$

meshedness = 0.3

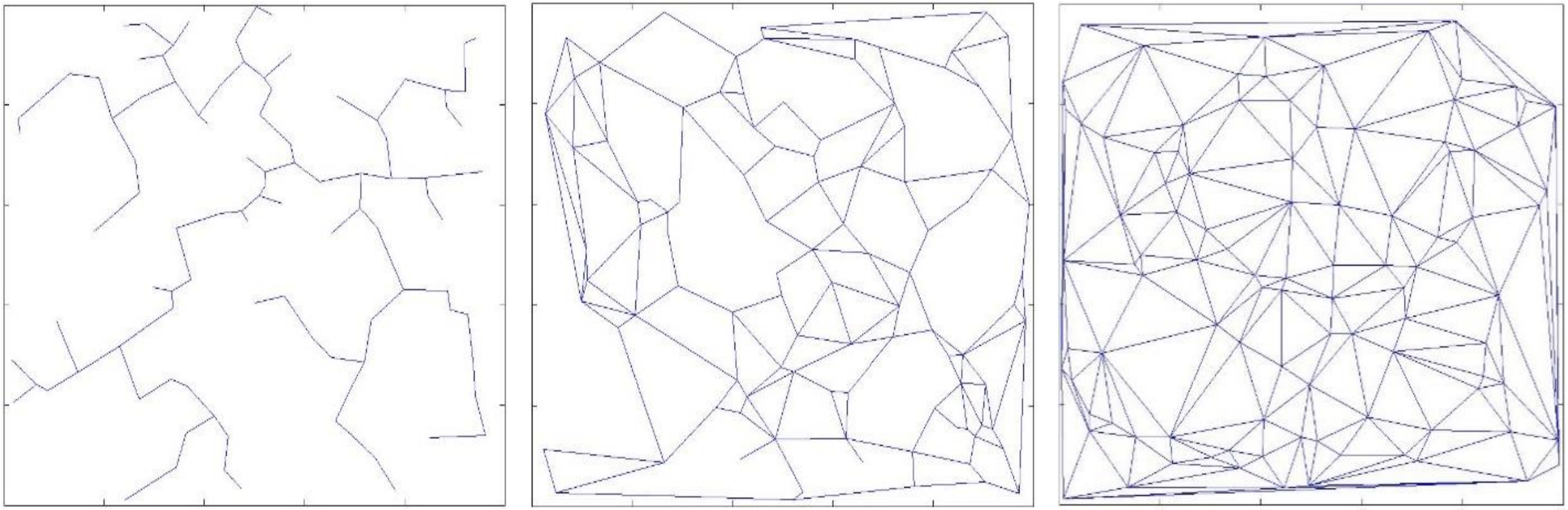
Network Size



Network Size



Network Connectivity [MST -> Delaunay]

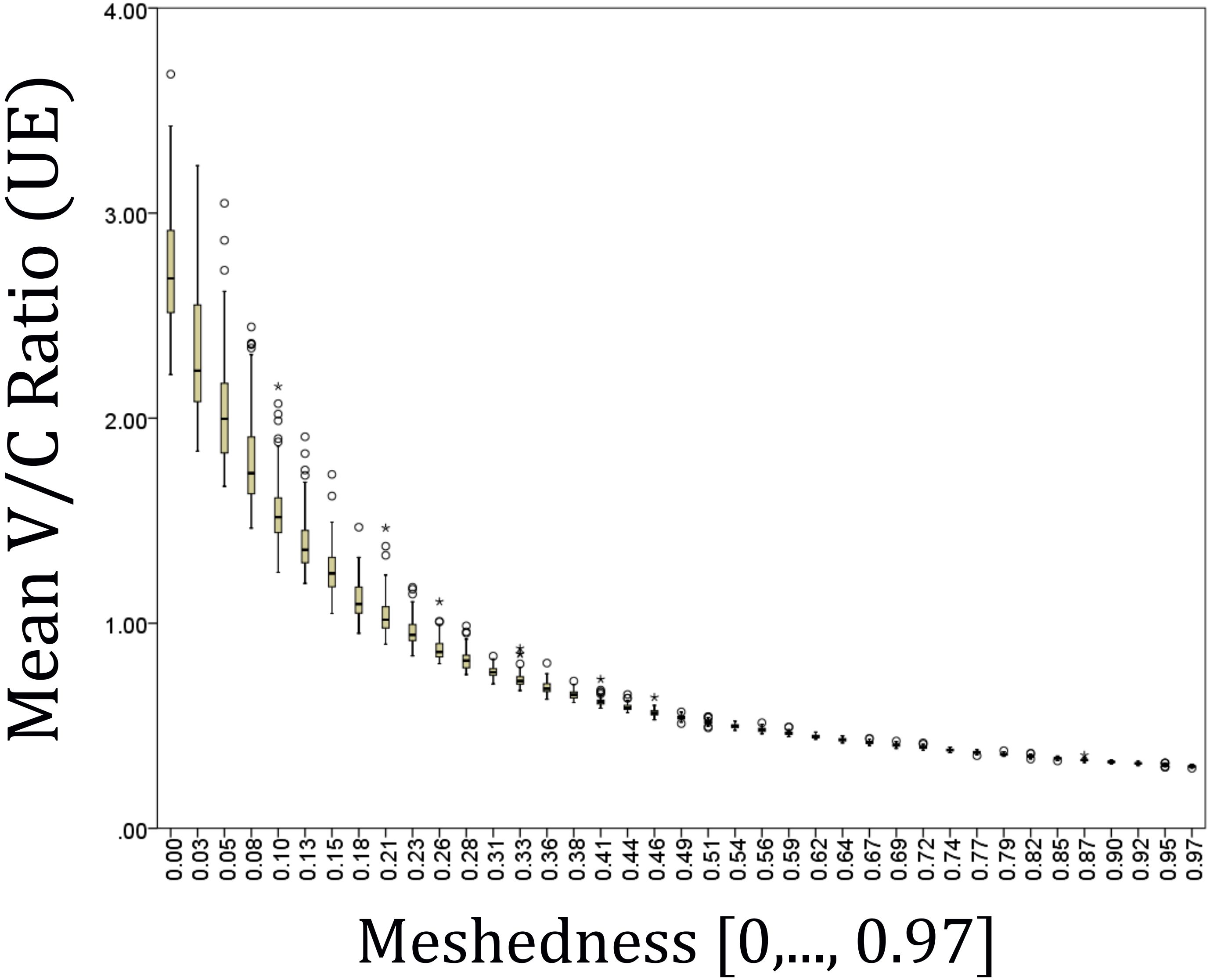


$\mathrm{M} = 0$ $\longrightarrow$ $\mathrm{M} = 0.97$

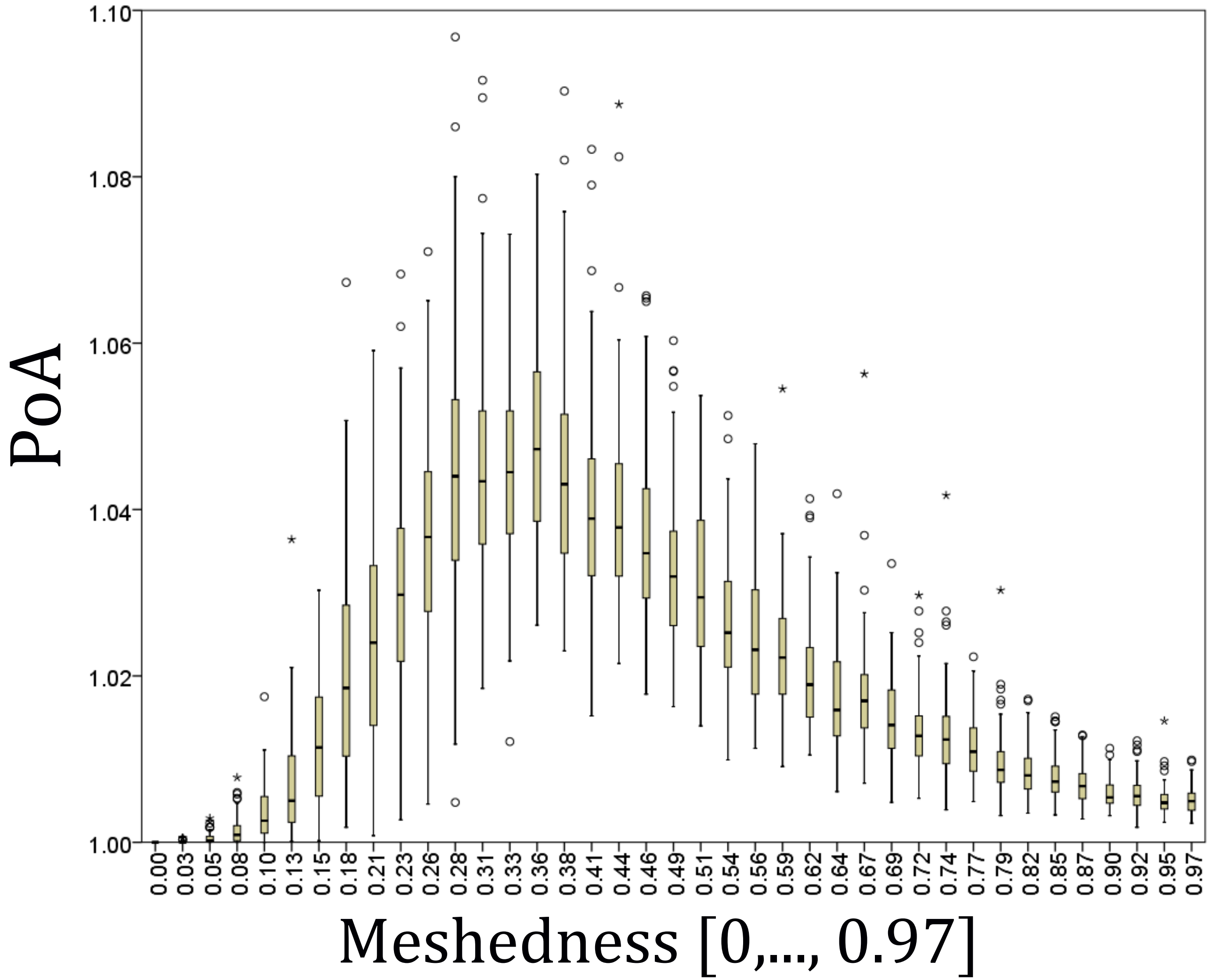
d_{min} = 0.05, A = 6.25 $\rho_{dem} = 4350$

$\mathrm{n} = 100 \quad \rho_n = 16$

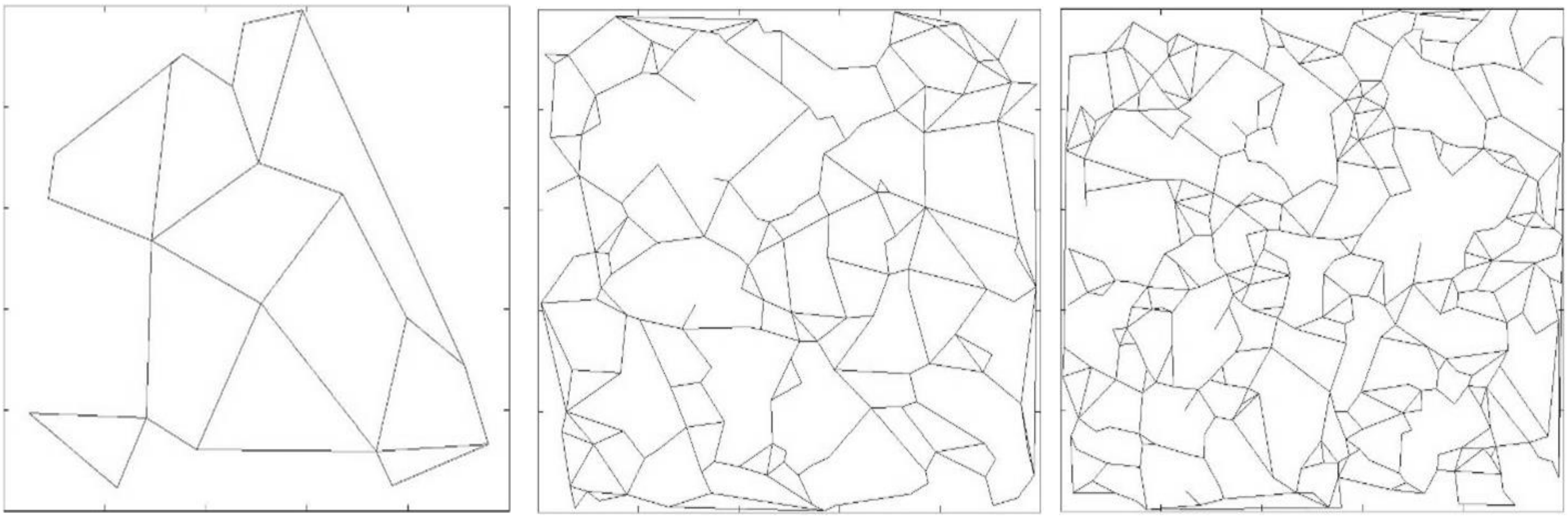
Network Size



Network Size



Network Density [node density increasing]

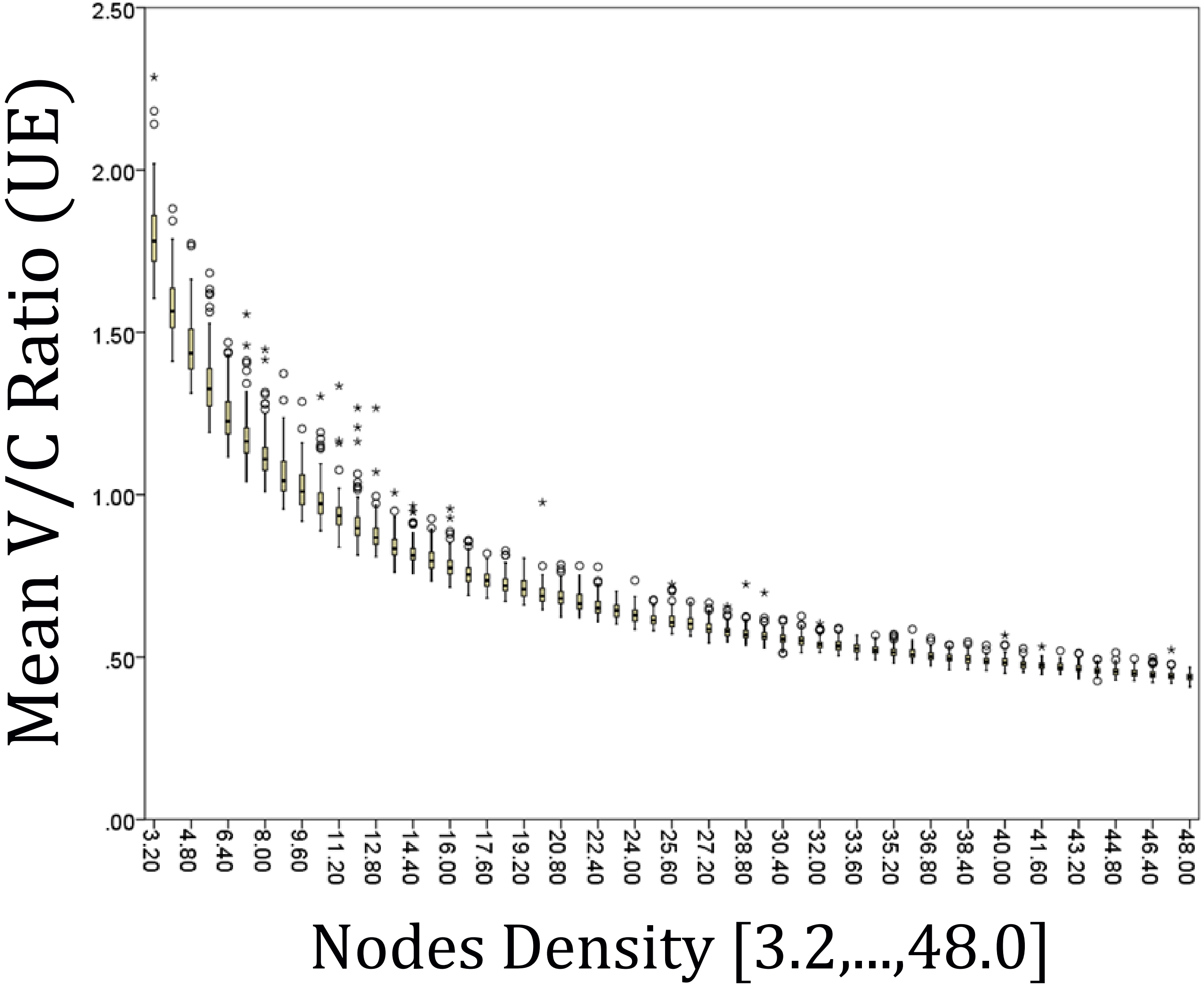


$\rho_n = 3.2 \longrightarrow \rho_n = 48$

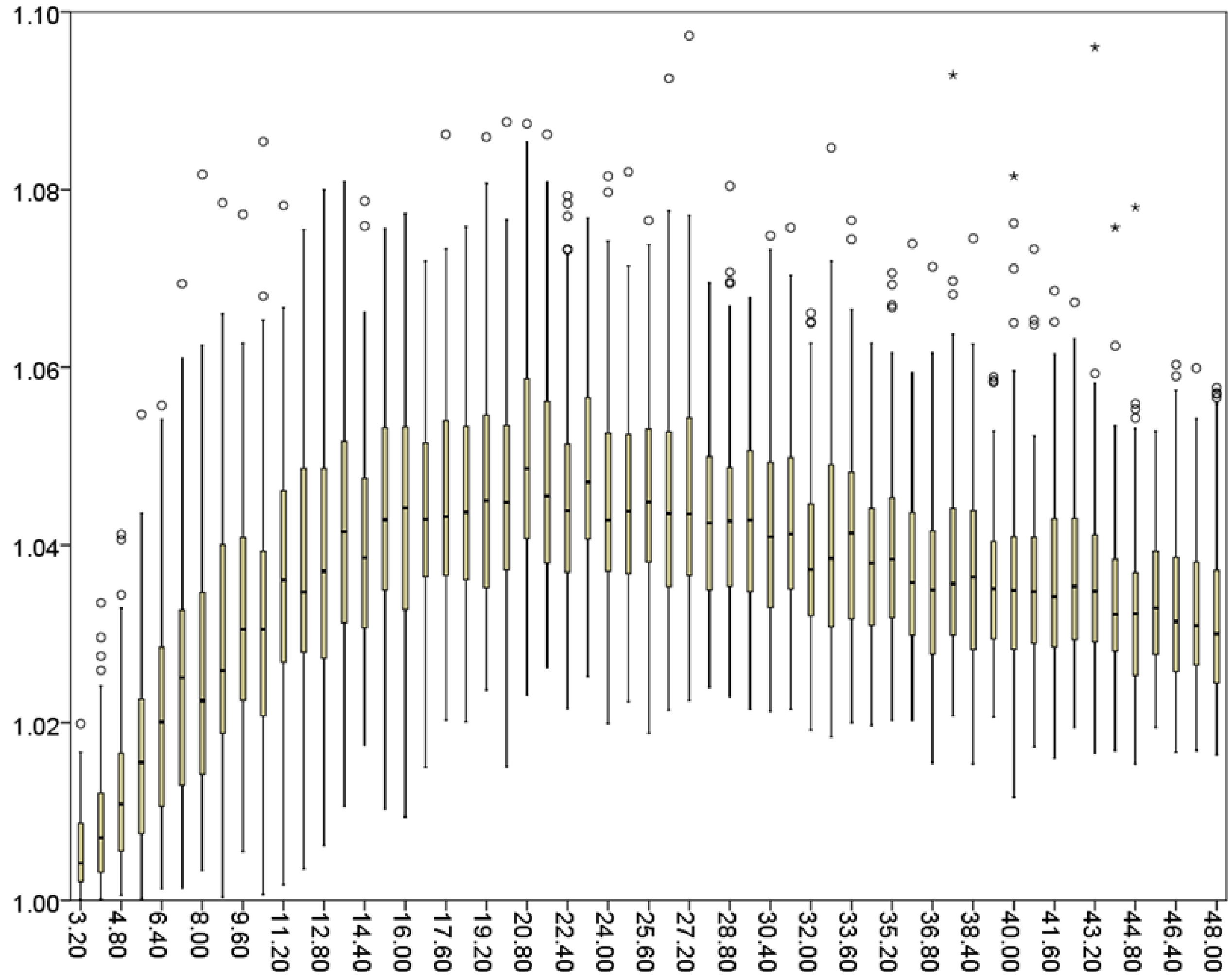
d_{min} = 0.05, A = 6.25 $\rho_{dem} = 4350$

Meshedness = 0.3

Network Size



Network Size



PoA

Nodes Density [3.2,...,48.0]

Conclusions/Questions

- Proposed ***ensemble analysis*** as an experimental approach (~ transport network science)
- Based on synthetic networks
  - Needs thought on what “realistic” means
  - Needs work on network generation
- Look at other performance measures (not just PoA).

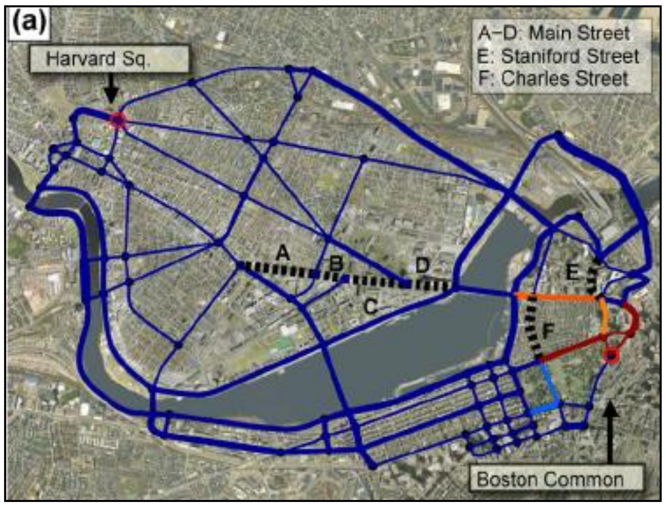
- See systematic relationship between PoA and the **quantity** of demand & **quantity** of supply.
- Have not yet clearly extracted the (relative) importance of the **configuration** of demand & supply.
- Interesting complication that the (active) network changes with demand.

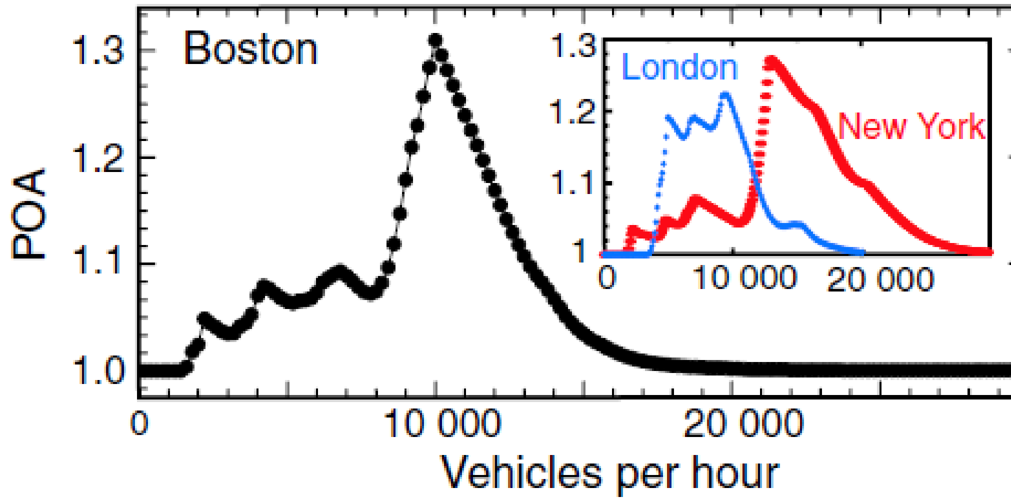
[redacted]

How does PoA depend on demand?

PoA In Real Networks

Price of Anarchy does vary with demand and supply structure in real networks:



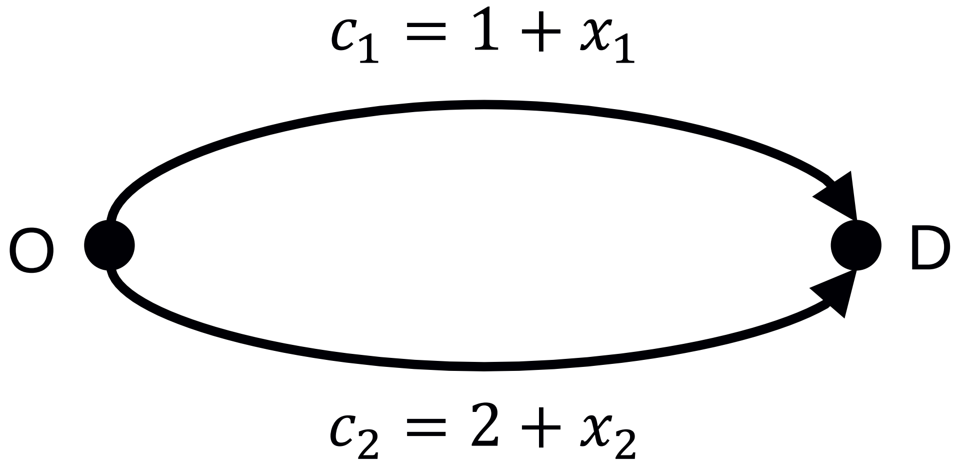


A single OD in each of 3 cities.

No explanation of what is going on.

Source: Youn et al. 2008.
Phys Rev Letters, 101.

Parallel Routes Network with Linear Costs

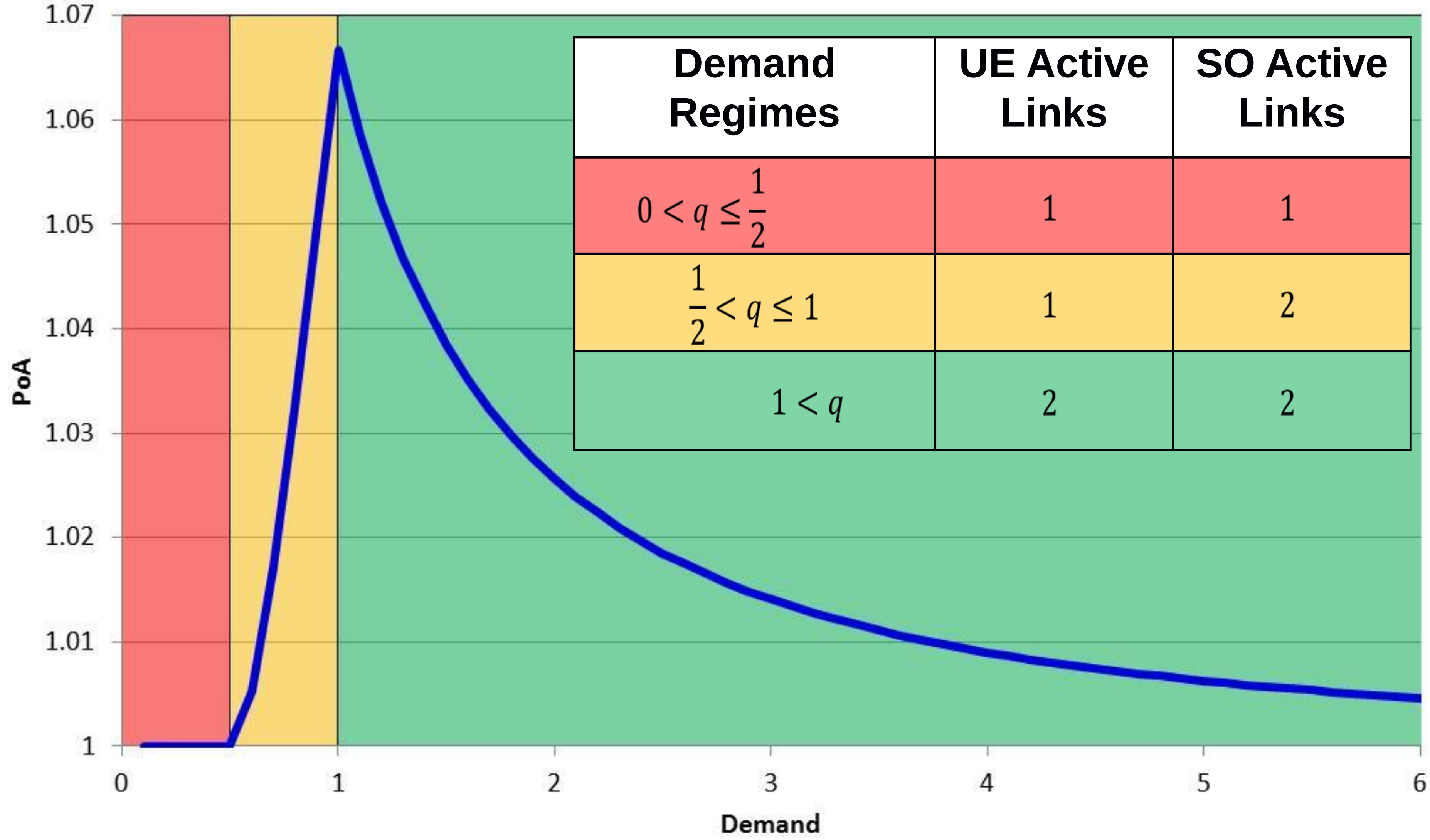


$$q_{OD} = \sum_{i=1}^{2} x_i$$

Initially, small demand, will only get flow on link 1

At some level of demand link 2 will "activate".

The Price of Anarchy against Demand for the 'Two Parallel Links' Network



Demand Regimes	UE Active Links	SO Active Links
$0 < q \leq \frac{1}{2}$	1	1
$\frac{1}{2} < q \leq 1$	1	2
$1 < q$	2	2

The Effects of UE and SO Route Activations

Under both UE and SO, the activation of a route leads to a slower rate of increase of TTC.

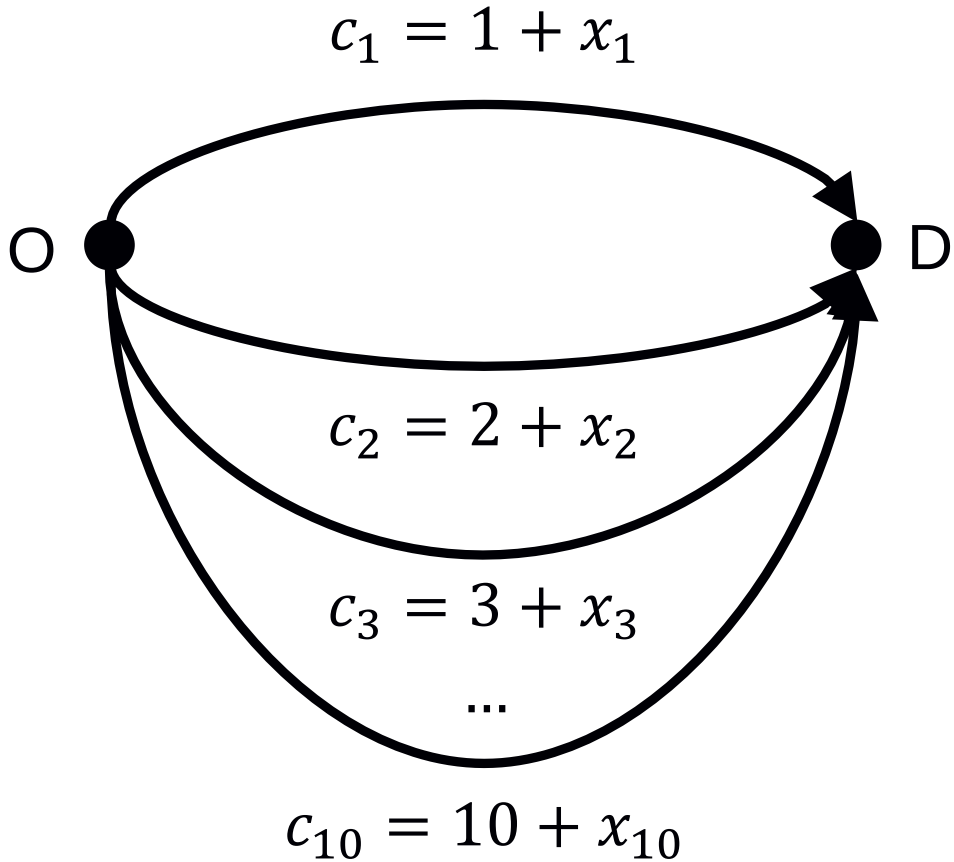
The effect of this on PoA differs depending on whether the activation occurs under UE or SO:

UE Activation ⇒ Numerator ↓ ⇒ PoA ↓

$$PoA = \frac{TTC^{UE}}{TTC^{SO}} = \frac{\sum_i x_i^{UE} c\left(x_i^{UE}\right)}{\sum_i x_i^{SO} c\left(x_i^{SO}\right)}$$

SO Activation ⇒ Denominator ↓ ⇒ PoA ↑

Ten Parallel Routes Network with Linear Costs

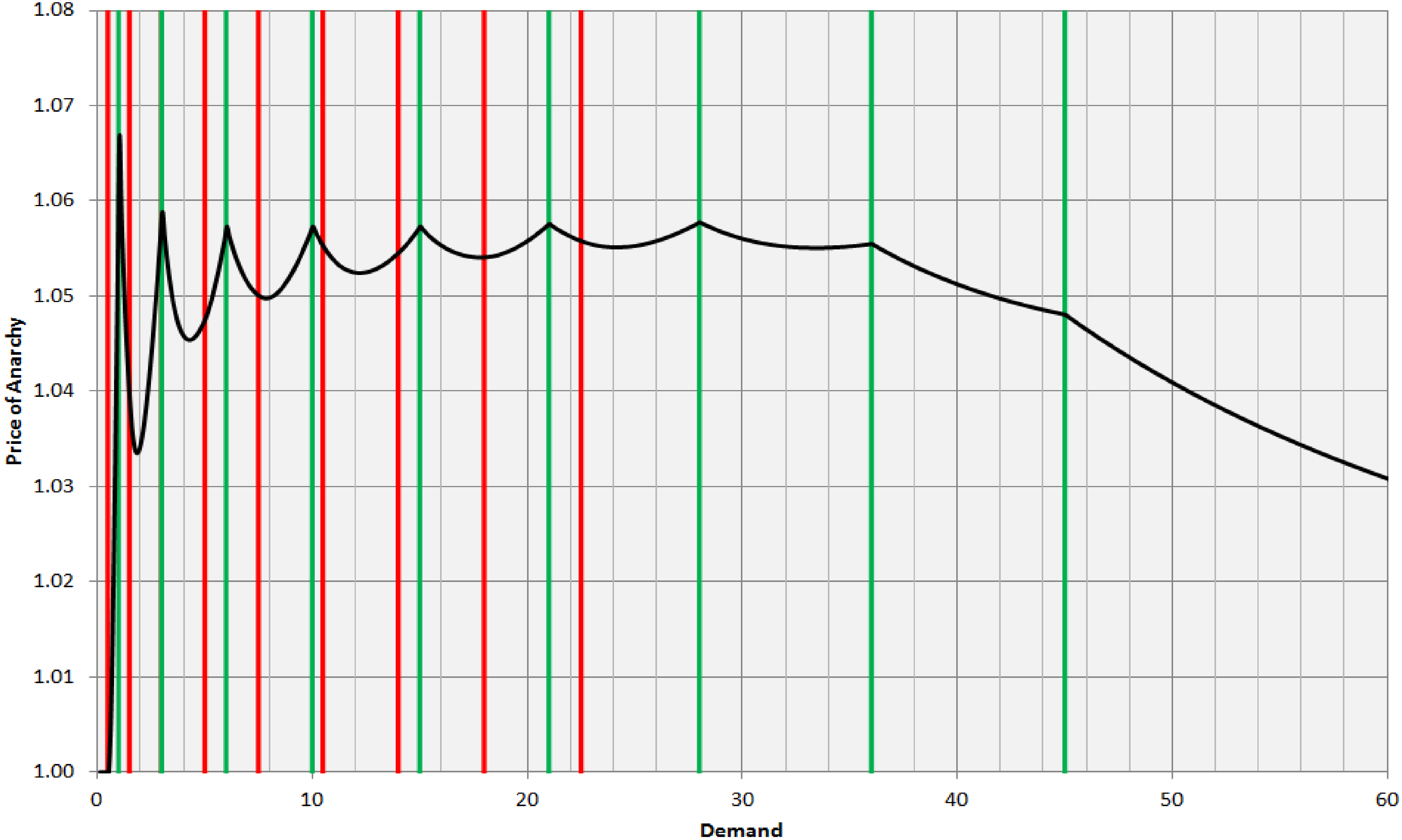


$$q_{OD} = \sum_{i=1}^{10} x_i$$

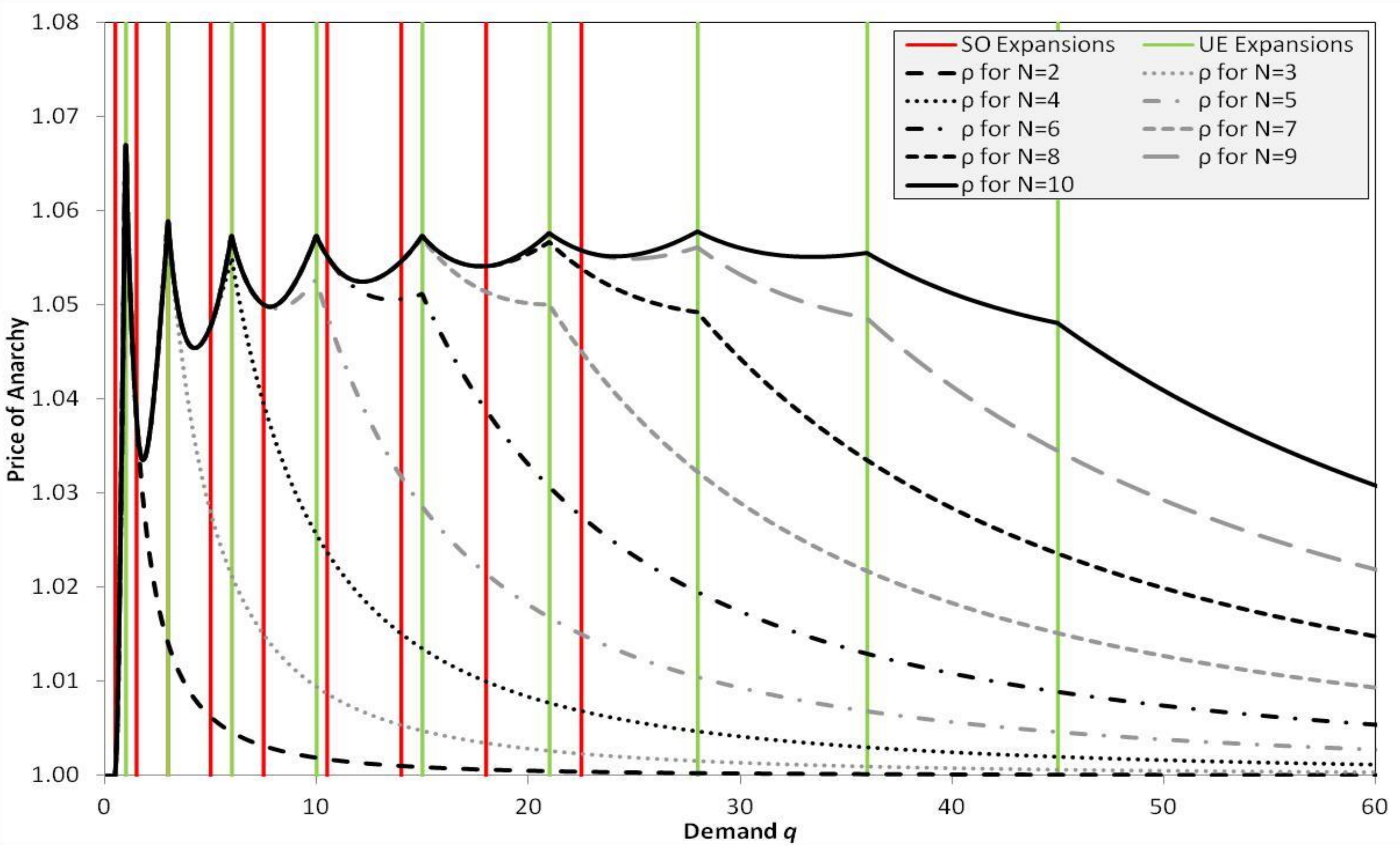
$$a_i = i \ \forall i$$

$$b_i = 1 \ \forall i$$

The Variation of the Price of Anarchy with Demand

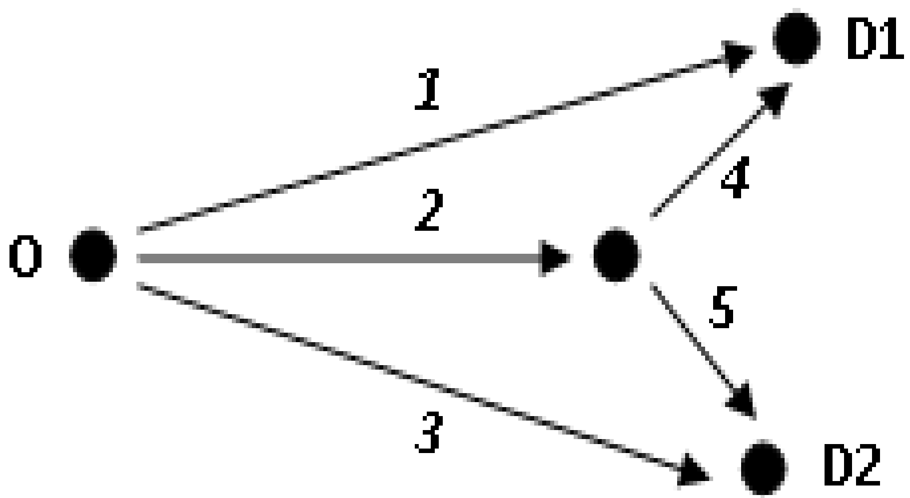


[redacted]



SO Expansions
UE Expansions
ρ for N=2
ρ for N=3
ρ for N=4
ρ for N=5
ρ for N=6
ρ for N=7
ρ for N=8
ρ for N=9
ρ for N=10
Price of Anarchy
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1.00
0
10
20
30
40
50
60
Demand q

Route DE-activation



$$q_{O-D2} = 1$$

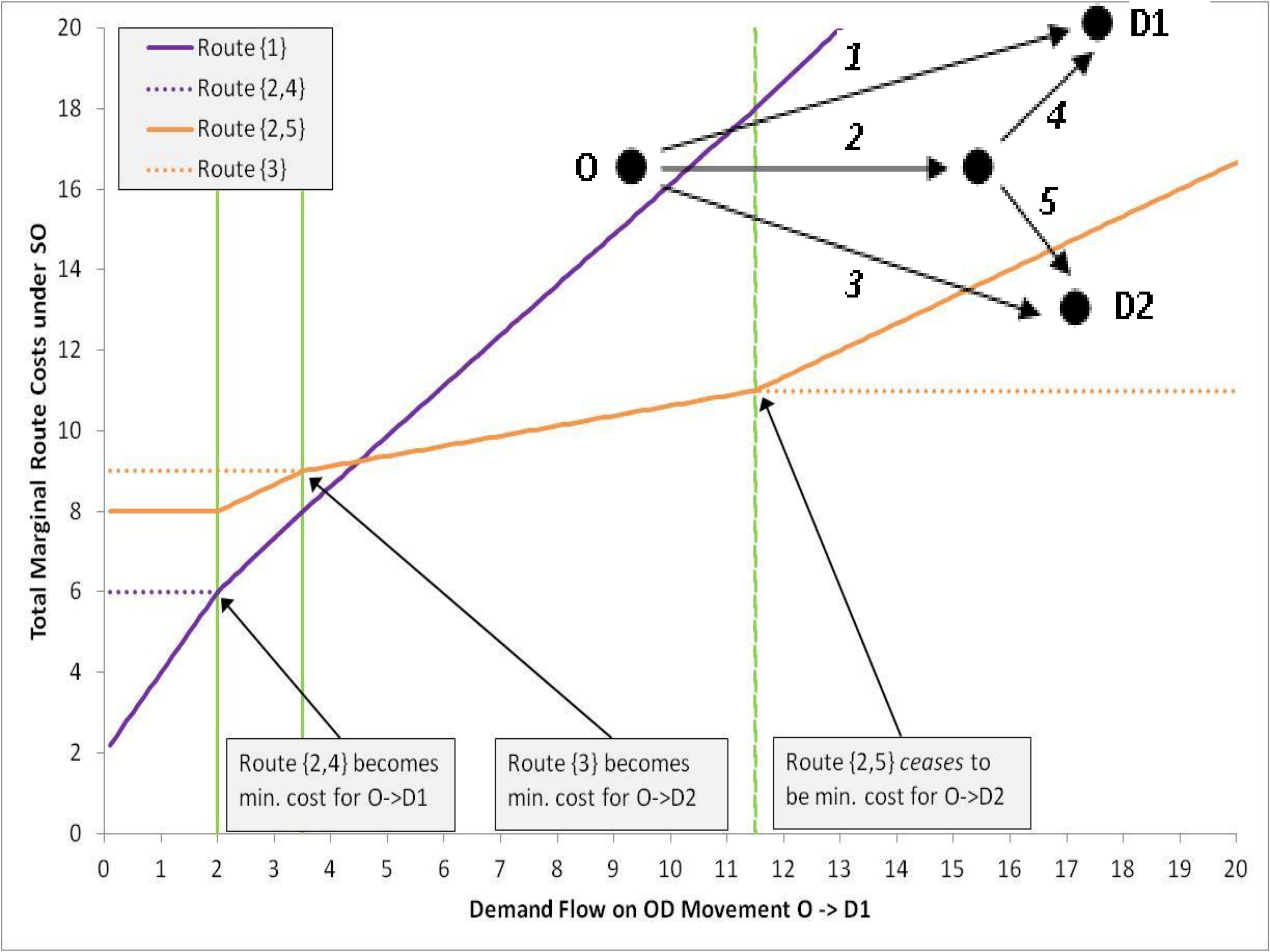
$$c_1 = 2 + x_1$$

$$c_2 = 3 + x_2$$

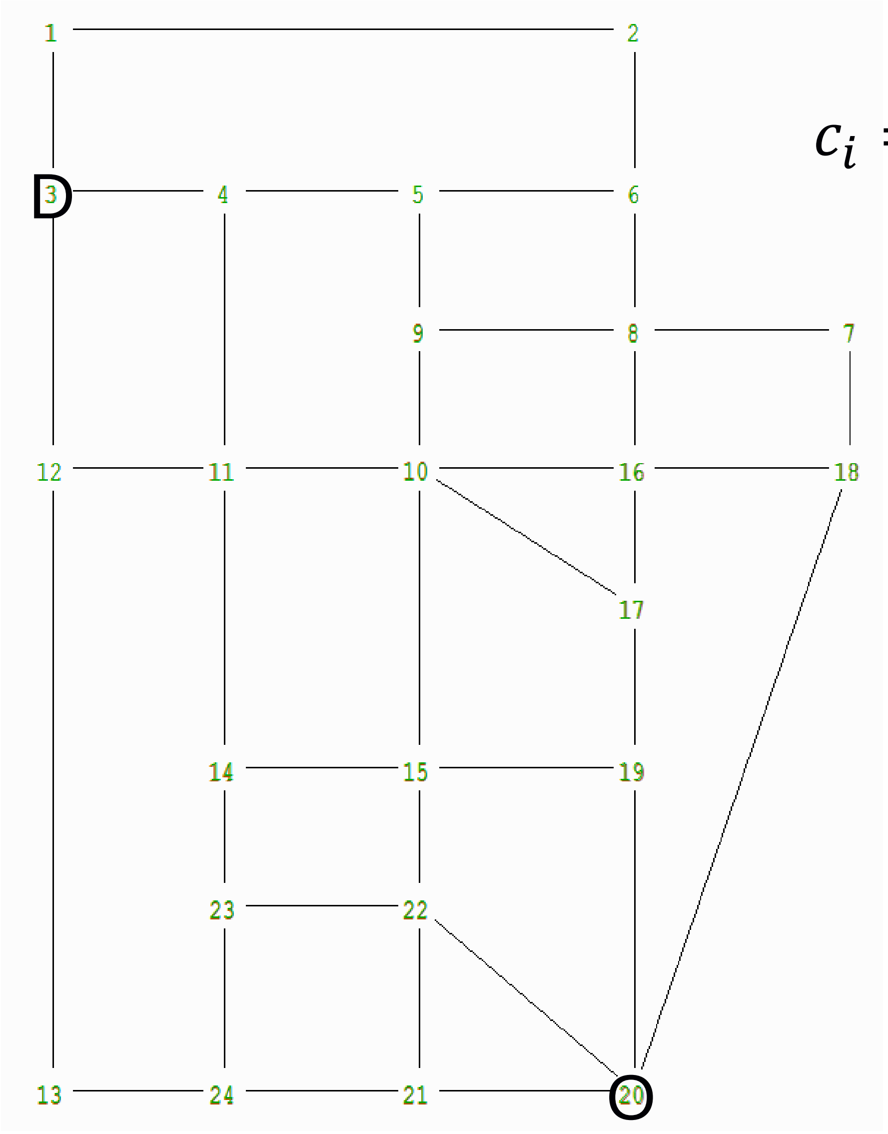
$$c_3 = 9 + x_3$$

$$c_4 = 1 + x_4$$

$$c_5 = 1 + x_5$$


Route {1}
Route {2,4}
Route {2,5}
Route {3}
O
D1
D2
1
2
3
4
5
Total Marginal Route Costs under SO
Demand Flow on OD Movement O -> D1
Route {2,4} becomes min. cost for O->D1
Route {3} becomes min. cost for O->D2
Route {2,5} *ceases* to be min. cost for O->D2

Sioux Falls network for a Single OD Pair



$$c_i = a_i + b_i x_i^4$$

$$\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & q_{20,3} & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$$

The Variation of the Price of Anarchy with Demand

Expansions and Contractions

Activations/Deactivations occur when the set of minimum OD cost routes changes (or minimum marginal total cost routes for SO).

Call activation/transition points η_{SO} and η_{UE}

Increasing demand can induce activation &/or deactivation of routes. [Note off, on, off, on]

PoA is differentiable except at UE transition points

Special Case

If all cost functions are $c_i = a_i + b_i x_i^\beta$

And link flow solutions are $x_i^{UE}(Q)$ $x_i^{SO}(Q)$

Then $$x_i^{SO}\left(\frac{Q}{\sqrt[\beta]{\beta+1}}\right) = \frac{1}{\sqrt[\beta]{\beta+1}} x_i^{UE}(Q)$$

And hence $$\eta_{SO} = \frac{1}{\sqrt[\beta]{\beta+1}} \eta_{UE}$$

Summary

- The variation of PoA with travel demand is due to *activation* of routes at *different* levels of demand under UE and SO.
- We have analysis of the types and nature of the “peaks” and “troughs”.

Questions?

Can we define canonical/reference networks?

Can we compare real network performance with a 'reference' network?

What are the limits of such 'ideal' transport systems?

- properties of "1 sq km of network"
- lattices and symmetric networks?
- demand and supply conditions
- vulnerability, resilience, redundancy

Connection to aggregation and network-free models.

Thank you.

Questions?