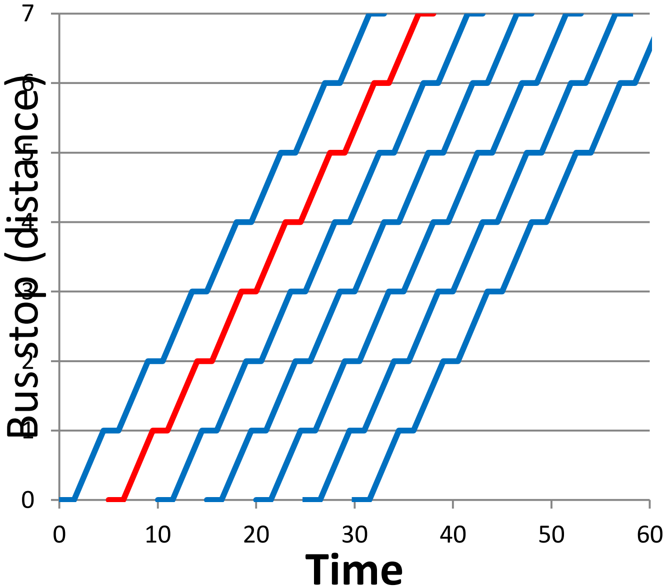
Smart transit systems for even smarter travellers

Jan-Dirk Schmöcker
schmoecker@trans.kuciv.kyoto-u.ac.jp

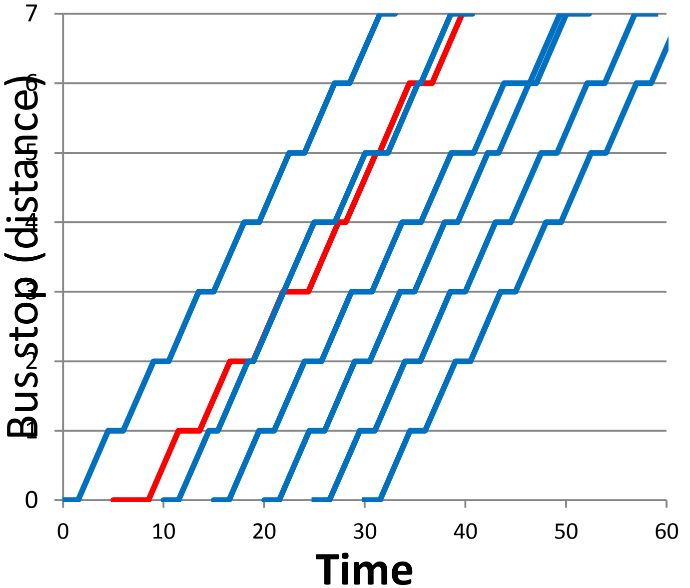
Overview

- Part 1: Single line, bus bunching
- Part 2a: Transit Route choice as game
- Part 2b: Notes on extension to all choices from O to D
- Conclusions/ current work

Bus bunching



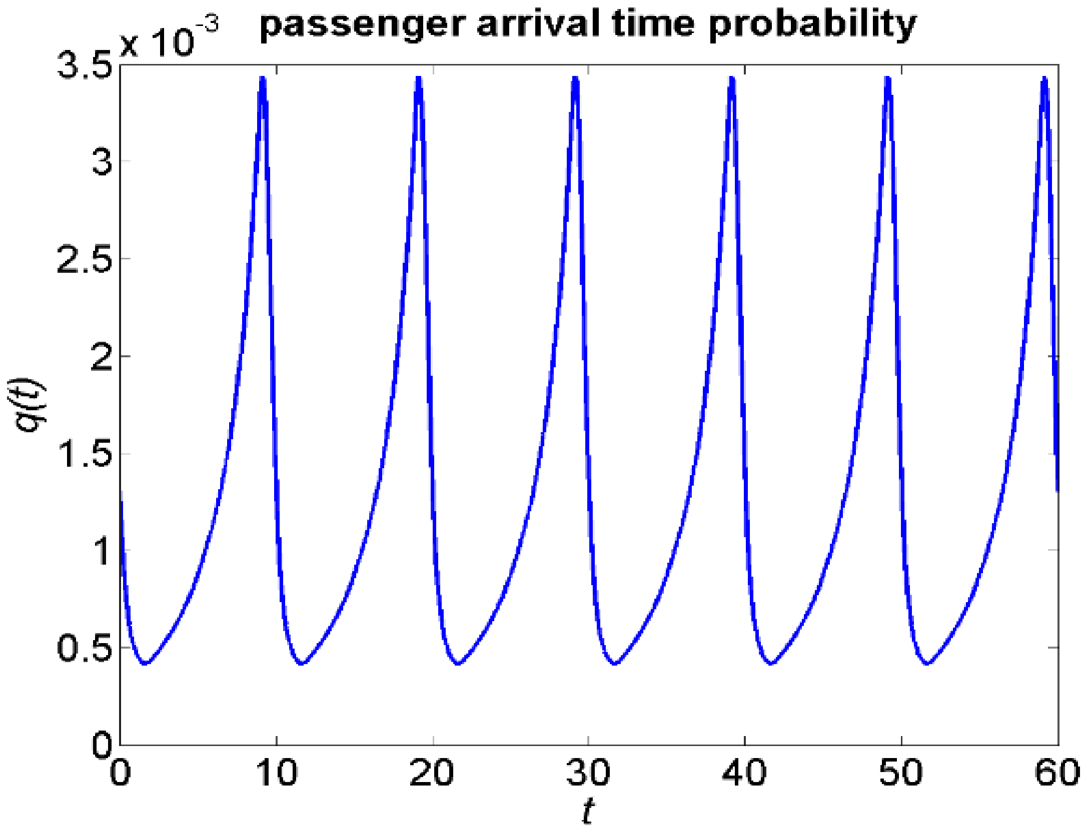
Without delay



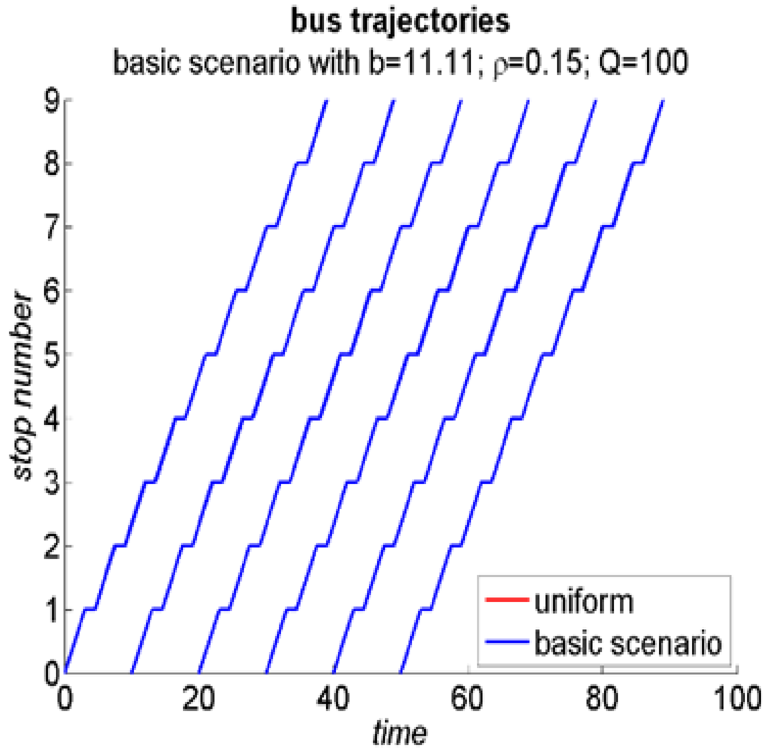
Initial delay for 2nd bus at stop 0

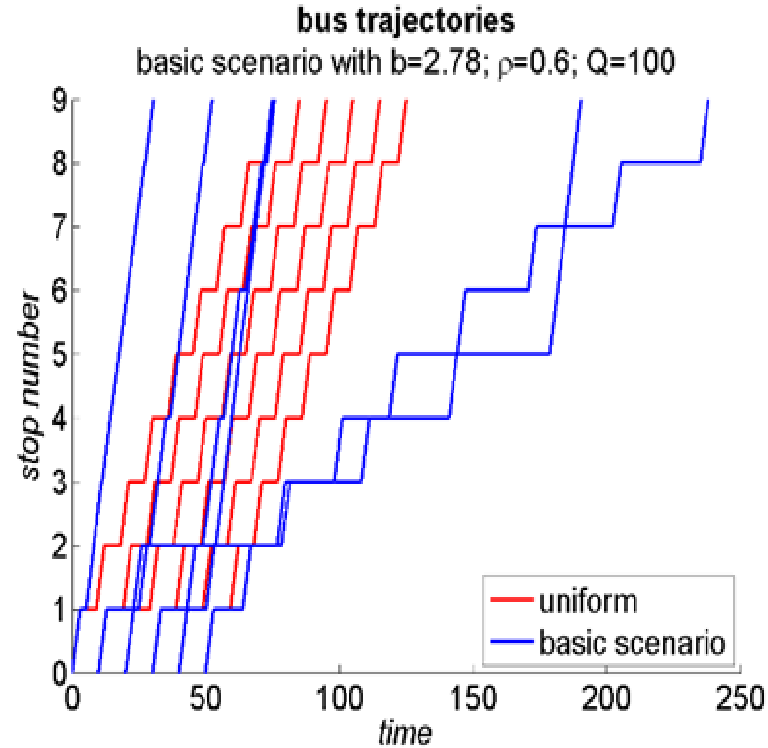
Bus bunching

Likely passenger stop arrival patterns with RTI



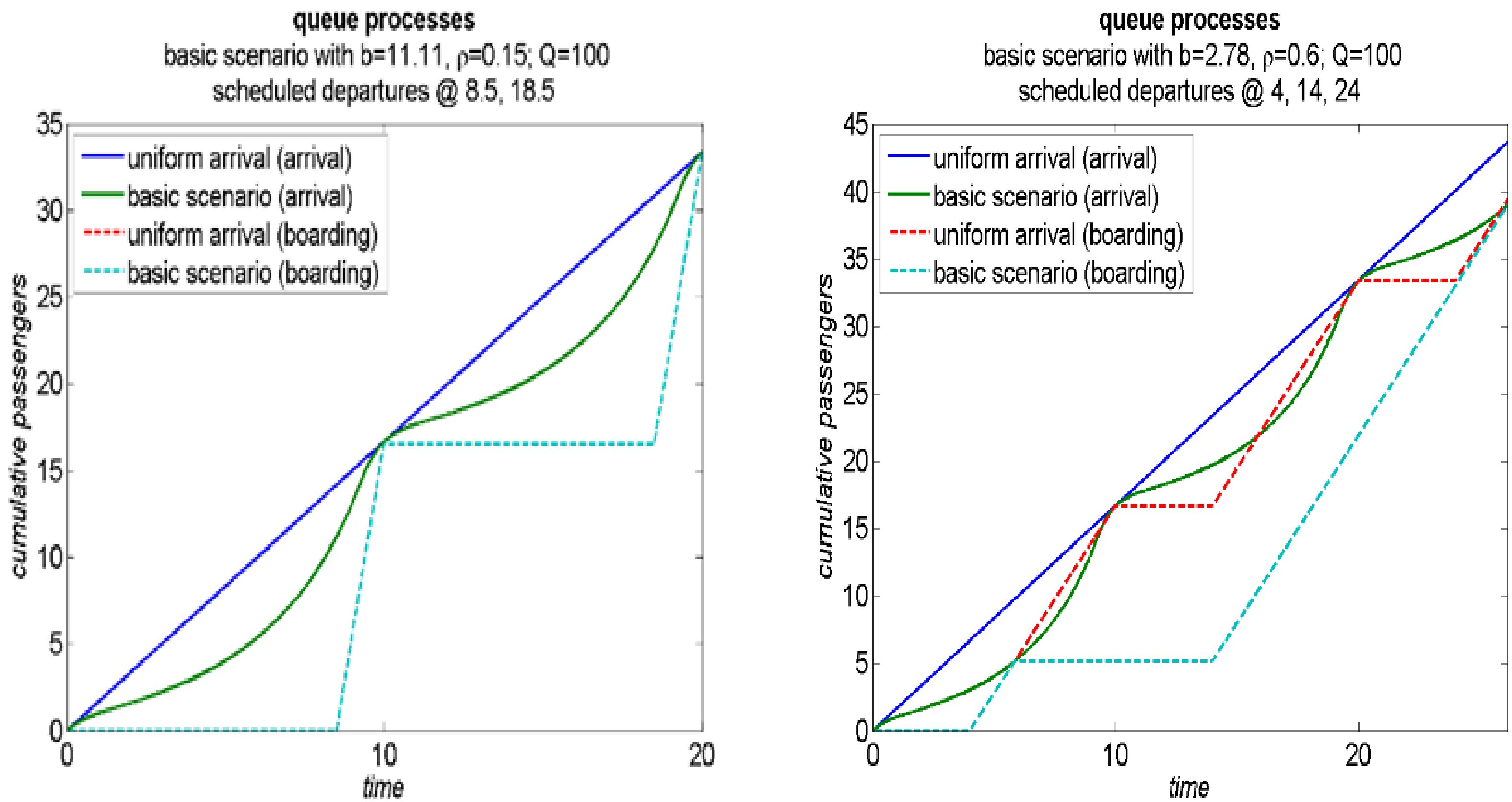
Influence of boarding rate on bunching





If the boarding rate is low, the service can be severely disrupted even without exogenous delays

Influence of boarding rate on bunching

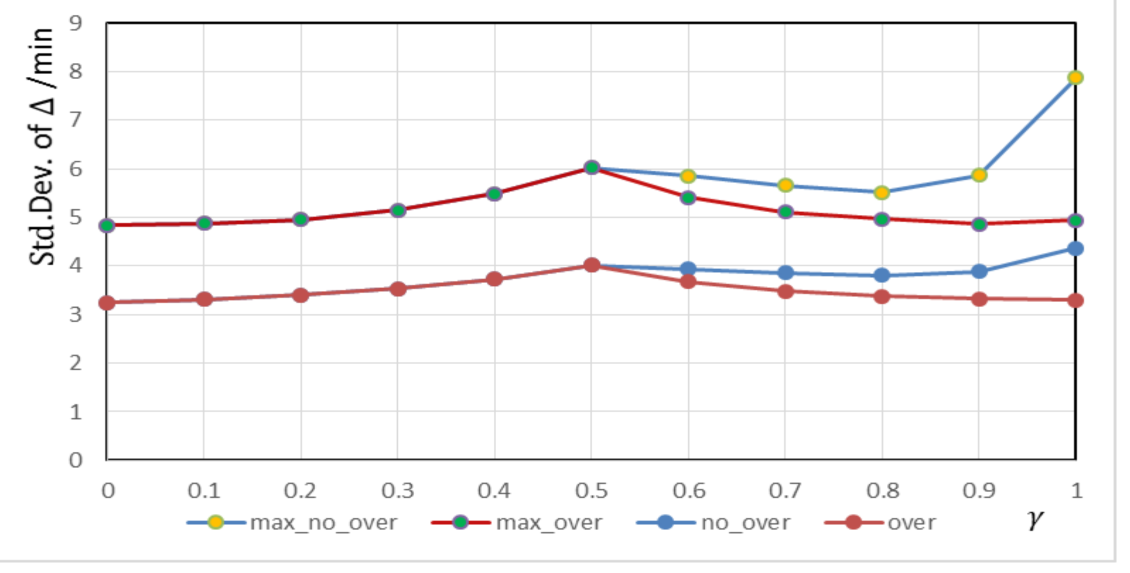


Conclusion:

- RTI “disturbs” passenger arrival patterns
- This means the system can be much easier perturbed - Even without exogenous delay
- Holding strategies become even more important

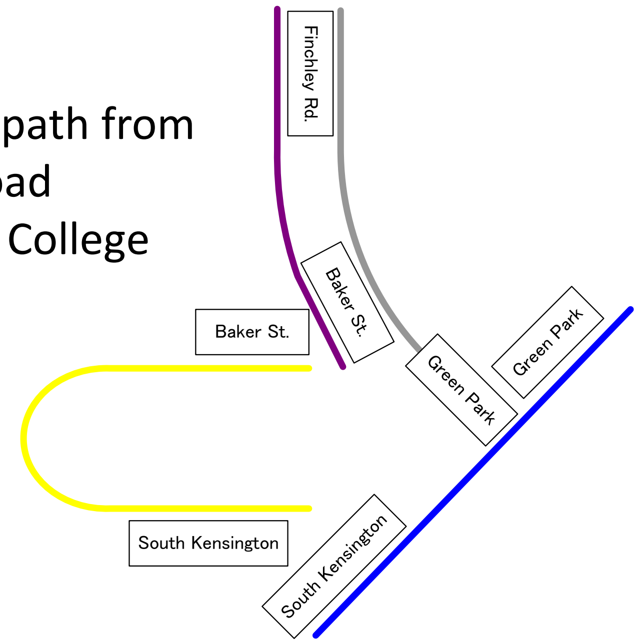
p.s. on control strategies

- All holding strategies introduce delays
- One control strategy is “the unfriendly bus driver”: *Go to the back bus, I am leaving.*

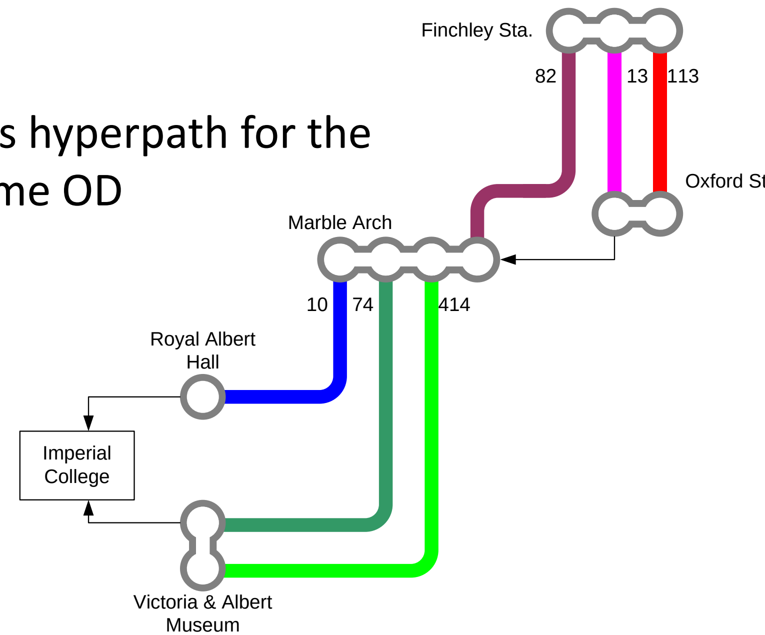


Part 2: Route choice in transit networks

Tube hyperpath from Finchley Road to Imperial College London



Bus hyperpath for the same OD



- Choosing a hyperpath consists of two steps
  - *Defining a set of paths*
  - *Defining the selection criteria* of a specific *path*

Spiess and Florian "Optimal strategies"

- Spiess and Florian (1989) proposed that passengers board the first line among a set of attractive lines at a boarding node *i.*
- Finding the optimal path set can be presented as a linear program where the objective is to find the strategy that minimises the *expected* waiting time.

$$p_a(A_i^+) = \frac{f_a}{\sum_{a \in A_i^+} f_a} \qquad w(A_i^+) = \frac{\alpha}{\sum_{a \in A_i^+} f_a}$$

Games and route choice

- “Hyperpaths” have also been applied in other contexts
- Risk-averse assignment leads to the creation of a set of paths in order to minimise the maximum travel cost
- Route choice as a game against single or multiple “demons” have been introduced to find worst case scenarios.
  - Bell (2000): Router vs. demon to find critical links
  - Cassir and Bell (2000) : Extension to multiple travellers
  - Cassir et al (2003): Tree spoiler to investigate reliability of specific Ods
  - Szeto et al (2007): Extension to multiple (independent) demons

Proposition: S&F = game

- The risk-averse traveller, who *fears* a maximum delay of $d_a = 1/f_a$ on any link should use the S &F path split probabilities, independent of the travel time on any downstream link c_a, and include all links that are not dominated by any other link.

- Note the difference in interpretation:
  - S&F : $1/f_a$ is the expected waiting time
  - Here : $1/f_a$ is the maximum link delay

Proof (1)

- The risk averse traveller fears that a line might be delayed by up to d_a, *this can be described as a game with following pay-off matrix:*

	q_1	q_2	…	q_k	…	q_n
p_1	$u_1 + d_1$	u_1	…	u_1	…	u_1
p_2	u_2	$u_2 + d_2$	…	u_2	…	u_2
…	…	…	…	…	…	…
p_k	u_k	u_k	…	$u_k + d_k$	…	u_k
…	…	…	…	…	…	…
p_n	u_n	u_n	…	u_n	…	$u_n + d_n$

- *..leading to following optimisation problem:*

$$\text{Min}_{\boldsymbol{p}}\ \text{Max}_{\boldsymbol{q}} \sum_{a \in A_i^+} u_a p_a + q_a p_a d_a$$

Proof (2)

- The travellers is hence to choose a (mixed) strategy **p** that minimises his feared cost of travel λ_i.

Min λ_i so that

$$
\begin{array}{lllllll}
p_1(u_1 + d_1) & + p_2 u_2 & + \dots & + p_k u_k & = & g_1 & \leq \lambda_i \\
p_1 u_1 & + p_2(u_2 + d_2) & + \dots & + p_k u_k & = & g_2 & \leq \lambda_i \\
\dots & + \dots & + \dots & + \dots & = & \dots & \leq \lambda_i \\
p_1 u_1 & + p_2 u_2 & + \dots & + p_k(u_k + d_k) & = & g_k & \leq \lambda_i
\end{array}
$$

$$p_1 + p_2 + \dots + p_k = 1$$

$$p_i > 0 \ \forall i = 1, \dots, k$$

- Following the expected value principle at the saddle point the costs of all used strategies will be equal.

Proof of proposition 1 (3)

- …hence solving the set of equations wlog for p_1 leads to

  $p_a = p_1 (d_1/d_a) \; \forall a=2,\ldots,k$

and

$$p_1 + p_1 (d_1/d_2) + p_1 (d_1/d_3) + \ldots + p_1 (d_1/d_k) = 1$$

- Solving for p_1 leads to:

$$p_1 = \frac{\frac{1}{d_1}}{\sum_{a=1,..,k} \frac{1}{d_a}}$$

qed

Further properties of this zero-sum game

- With p_a determined it follows for the expected game value:

$$g = \left(u_1 + \frac{1}{f_1} \right) \frac{f_1}{\sum_i f_i} + u_2 \frac{f_2}{\sum_i f_i} + \ldots + s_n \frac{f_n}{\sum_i f_i} = \frac{1 + \sum_i f_i u_i}{\sum_i f_i}$$

- which is also equivalent to the S&F solution.

- In the same way as for the path split probabilities the attack probabilities q_a can be found for the Nash equilibrium solution.

Equivalence of S&F linear program with “Multiple local demon game”

- Spiess and Florian showed that following LP determines the optimal hyperpath (with assumptions as before)

$$Min_{\mathbf{p},\mathbf{w}} \sum_{a\in A} c_a p_a + \sum_{i\in I} w_i$$

Subject to

$$\sum_{a\in A_i^+} p_a - \sum_{a\in A_i^-} p_a = g_i$$

$$p_a d_a \le w_i$$

$$p_a \ge 0$$

- The corresponding Lagrangian function for this LP is:

$$L(\mathbf{p},\mathbf{w},\boldsymbol{\lambda},\mathbf{q}) = \sum_{a\in A} c_a p_a + \sum_{i\in I} w_i - \sum_{i\in I}\sum_{a\in A_i^+} q_a (w_i - p_a d_a) + \sum_{i\in I} \lambda_i (g_i - \sum_{a\in A_i^+} p_a + \sum_{a\in A_i^-} p_a)$$

Equivalence of S&F linear program with "Multiple local demon game" (2)

$$L(\mathbf{p},\mathbf{w},\boldsymbol{\lambda},\mathbf{q}) = \sum_{a\in A} c_a p_a + \sum_{i\in I} w_i - \sum_{i\in I}\sum_{a\in A_i^+} q_a (w_i - p_a d_a) + \sum_{i\in I} \lambda_i (g_i - \sum_{a\in A_i^+} p_a + \sum_{a\in A_i^-} p_a)$$

- The primary ↔ dual variables are **p** ↔ **λ** and **w** ↔ **q**
  - Ahuja et al (1993) show that the dual variable of link choice probabilities **p** can be interpreted as node potential (as in Proposition 1)
  - Interpretation of **q** *…to follow*
- L(**p**,**w**,**λ**,**q**) is
  - Minimised with regards to **p** and **w**
  - Maxmised with regards to **q** and **λ**
  - Subject to non-negativity conditions on **p** and **q**

Equivalence of S&F linear program with "Multiple local demon game" (3)

- The interpretation of **q** is clarified by the dual problem
- L(**p**,**w**,**λ**,**q**) can be transformed into

$$L(\mathbf{p},\mathbf{w},\boldsymbol{\lambda},\mathbf{q}) = \sum_{i \in I} \lambda_i g_i - \sum_{a=(i,j) \in A} p_a((\lambda_i - \lambda_j) - c_a - q_a d_a)$$
$$- \sum_{i \in I} w_i (\sum_{a \in A_i^+} q_a - 1)$$

- Meaning that the dual problem can be formulated as

$$Max \sum_{i \in I} \lambda_i g_i$$

Subject to

$$(\lambda_i - \lambda_j - q_a d_a) \le c_a$$
$$\sum_{a \in A_i^+} q_a - 1 = 0$$
$$q_a \ge 0$$

Demon Problem: Maximise node costs subject to $q_i = 1$ at each node

Equivalence of S&F linear program with "Multiple local demon game" (4)

- Further, L(**p**,**w**,**λ**,**q**) can be transformed into

$$Max_{\mathbf{q}} Min_{\mathbf{p}} \sum_{a \in A} (c_a p_a + q_a d_a p_a)$$

Subject to

$$\sum_{a \in A_i^+} p_a - \sum_{a \in A_i^-} p_a = g_i$$

$$1 = \sum_{a \in A_i^+} q_a$$

$$p_a \geq 0$$

$$q_a \geq 0$$

- This MaxMin problem gives the mixed strategy Nash equilibrium of a zero sum, non-cooperative game between a network user endeavouring to minimise his travel cost and node specific demons aiming to penalise the traveller by imposing delays.

Comparison to single demon game

- As pointed out the single demon game has been of special interest in the network reliability literature.
- The S&F / Multiple local demon game can be transformed into the single demon game by a change in the demon constrained:

$$Max_{\mathbf{q}} Min_{\mathbf{p}} \sum_{a \in A} (c_a p_a + q_a d_a p_a)$$

Subject to

$$\sum_{a \in A_i^+} p_a - \sum_{a \in A_i^-} p_a = g_i$$

$$1 = \sum_{a \in A} q_a$$

$$p_a \geq 0$$

$$q_a \geq 0$$

$$a \in A_i^+$$

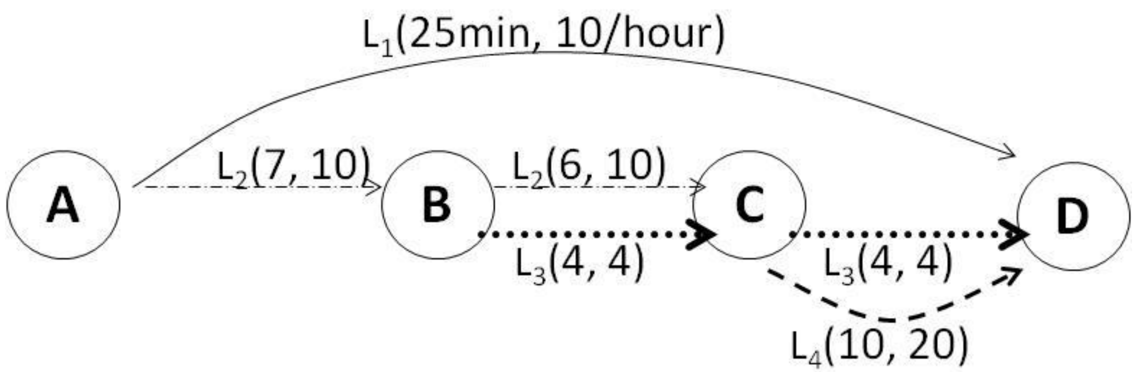
$$\downarrow$$

$$a \in A$$

Observation (can be proven)

- **Only the single demon game necessarily includes the shortest path**
  - In the multiple local demon game the traveller might avoid nodes with potentially large delays altogether.
  - In the single demon game the total resource of the demon is limited, meaning that the traveller should include (at least with a small probability) every potentially shortest path.

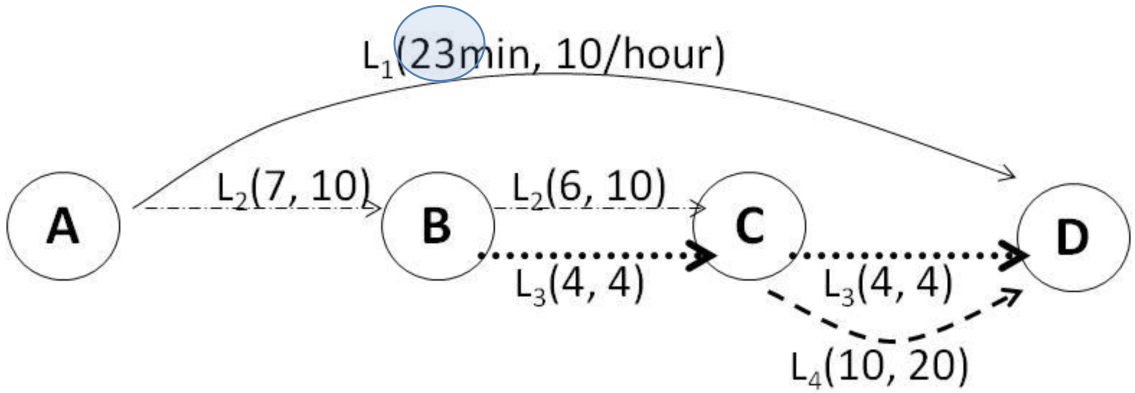
Numerical Example



	Multiple local demons		***Single demon***	
Link usage	p_a	q_a	p_a	q_a
L1	0.5	0.46	0	0
L2 (A-B)	0.5	0.54	1	0.07
L2(B-C)	0.5	*0*	0.6	0
L3(B-C)	0	*1*	0.4	0.53
L3(C-D)	0.08	0.5	0.8	0.40
L4	0.42	0.5	0.2	0
Game value/ feared travel cost	27.75 min		23.40 min	

- Shortest undelayed path L2(A-B) -> L3(B-C-D) only included in single demon game

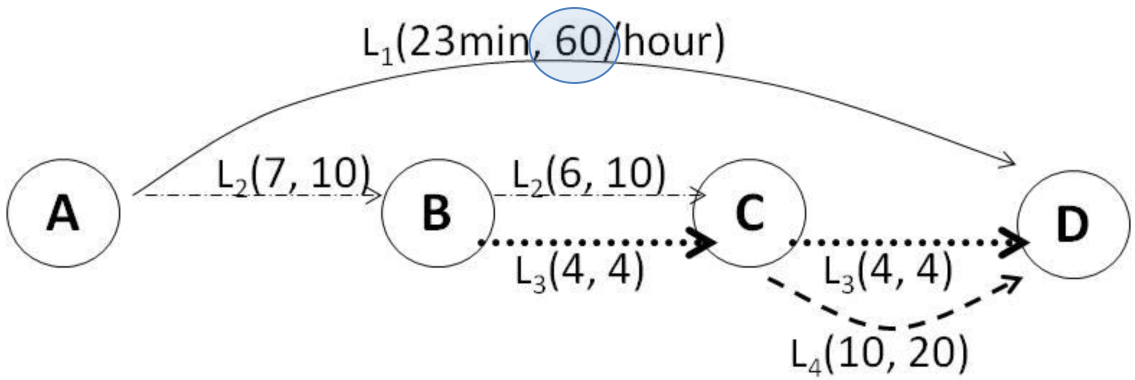
Numerical Example (2)



	Multiple local demons		*Single demon*	
Link usage	p_a	q_a	p_a	q_a
L1	0.50	0.63	0.5	0.03
L2 (A-B)	0.50	0.37	0.5	0.05
L2(B-C)	0.50	*0*	0.3	0
L3(B-C)	0	*1*	0.2	0.53
L3(C-D)	0.08	0.50	0.4	0.39
L4	0.42	0.50	0.1	0
Game value/ feared travel cost	26.75 min		23.20 min	

- Only **q** changes in MLDG
- L1 now also included in SDG

Numerical Example (3)



	Multiple local demons		***Single demon***	
Link usage	p_a	q_a	p_a	q_a
L1	1.00	1.00	0.86	0.06
L2 (A-B)	0	0	0.14	0.01
L2(B-C)	0	*0.33*	0.08	0
L3(B-C)	0	*0.67*	0.06	0.53
L3(C-D)	0	*0.5*	0.12	0.4
L4	0	*0.5*	0.03	0
Game value/ feared travel cost	24.00 min		23.06 min	

- Only single route in MLDG
- Multiple routes in SDG (as 24min > travel time on shortest path)

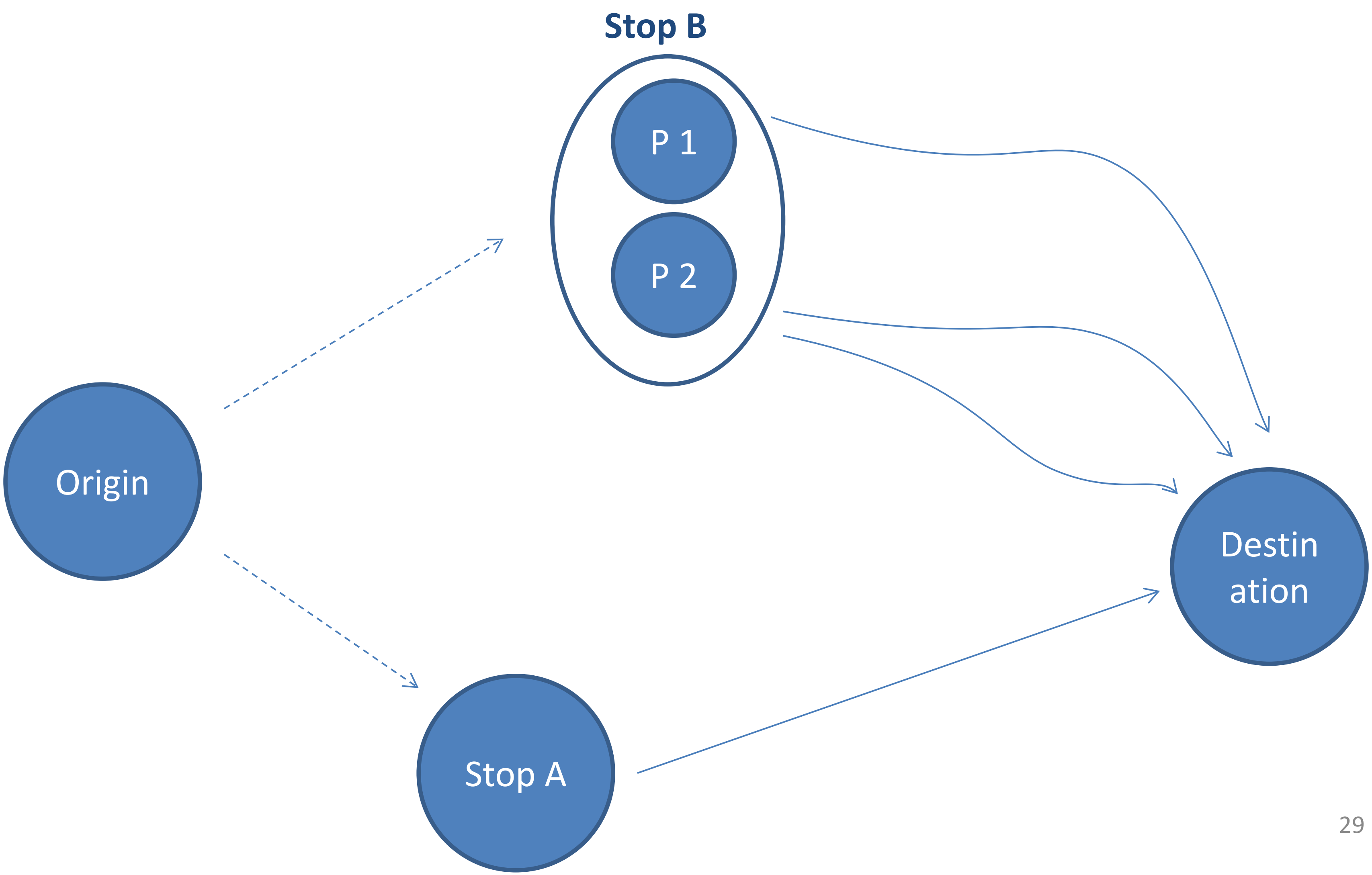
Conclusions

- The S&F hyperpath concept can be interpreted as a “multiple local demon game” (MLDG).
  - It is the route choice of an intelligent traveller fearing that “something can go wrong at each decision point”.
  - The dual variable **q** is an indicator for the link importance
- A smart transit system with RTI and line coordination can reduce the tavellers’ game to a single demon game.

Extension: Effects of information on trip stages

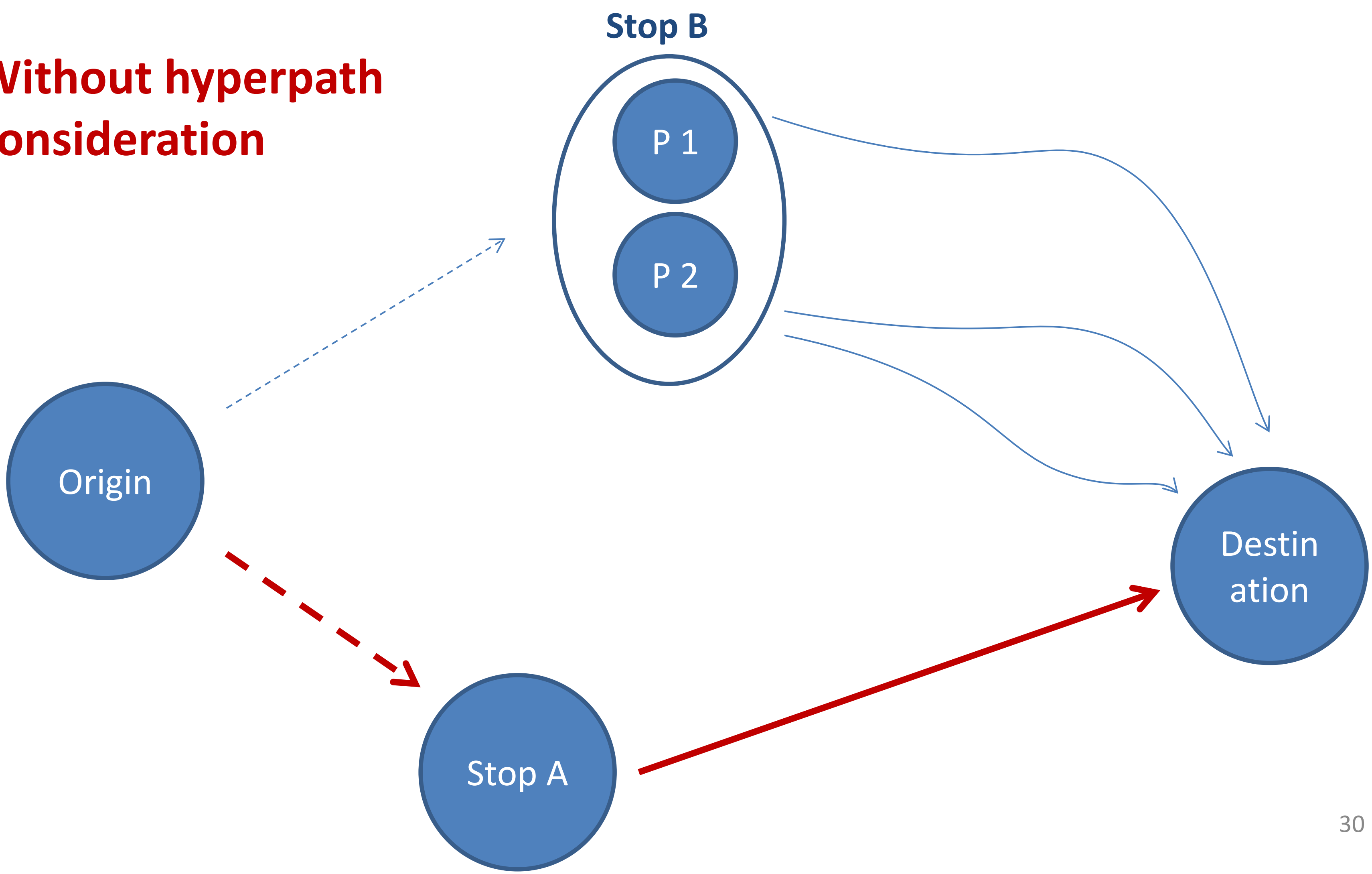
Travel stage	Uninformed, unfamiliar traveller	Commuter; Local dynamic information	Ubiquitous Info (Smart phone
Before departure	Look up timetable, select stop and dep. time	Select stop and dep. time, possibly complex alternatives and more complex strategies	
At stop	Wait for service	possibly consider more connections	+ possibly consider changing stop
On-Board	No decision	possibly adjust alighting point	+ possibly change dest.
At Destination	Stick to plan or make an effort to obtain new information		Possibly revise and co-ordinate plans; e.g. adjust pick-up

Network Example



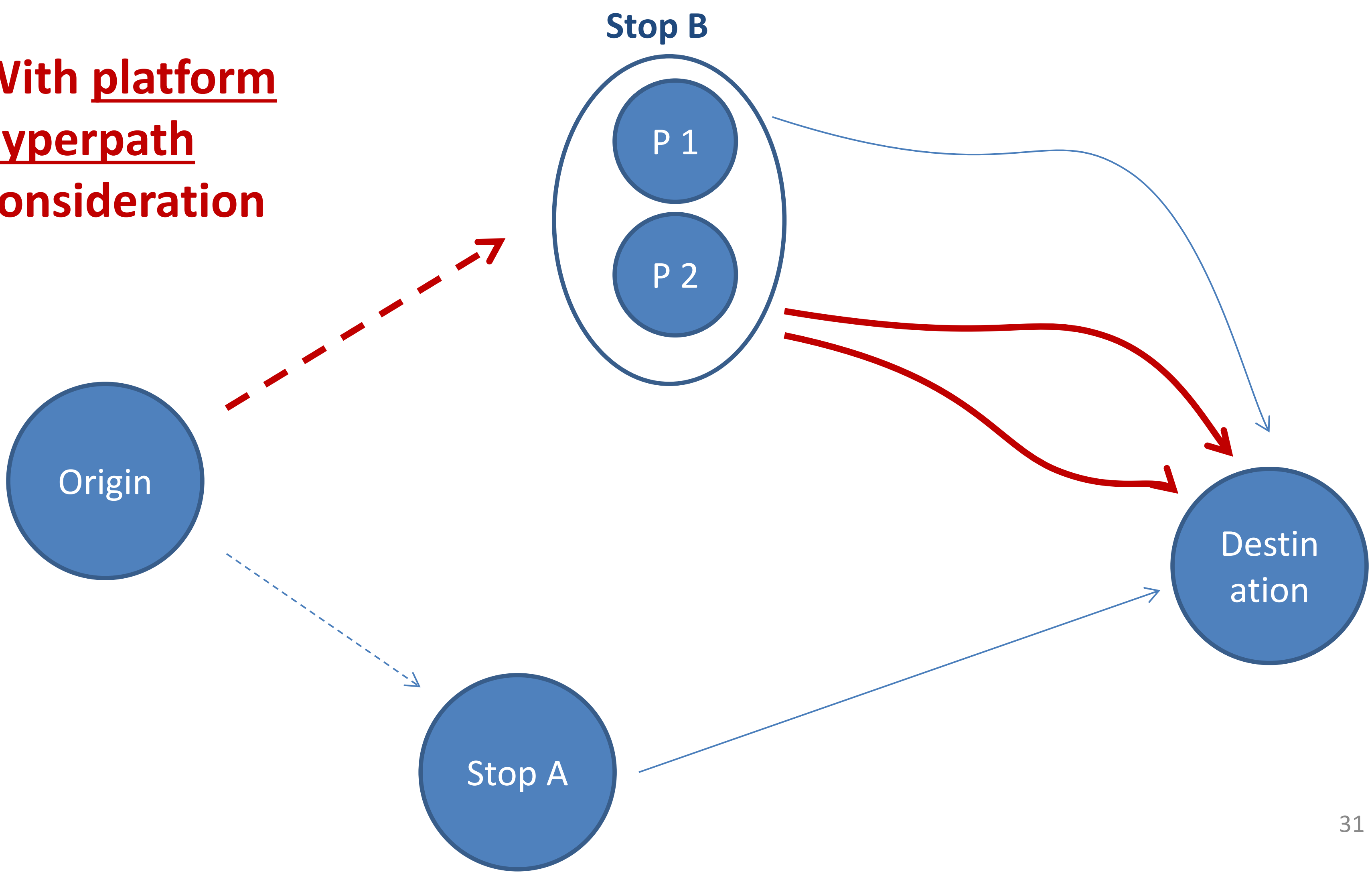
Network Example

Without hyperpath consideration



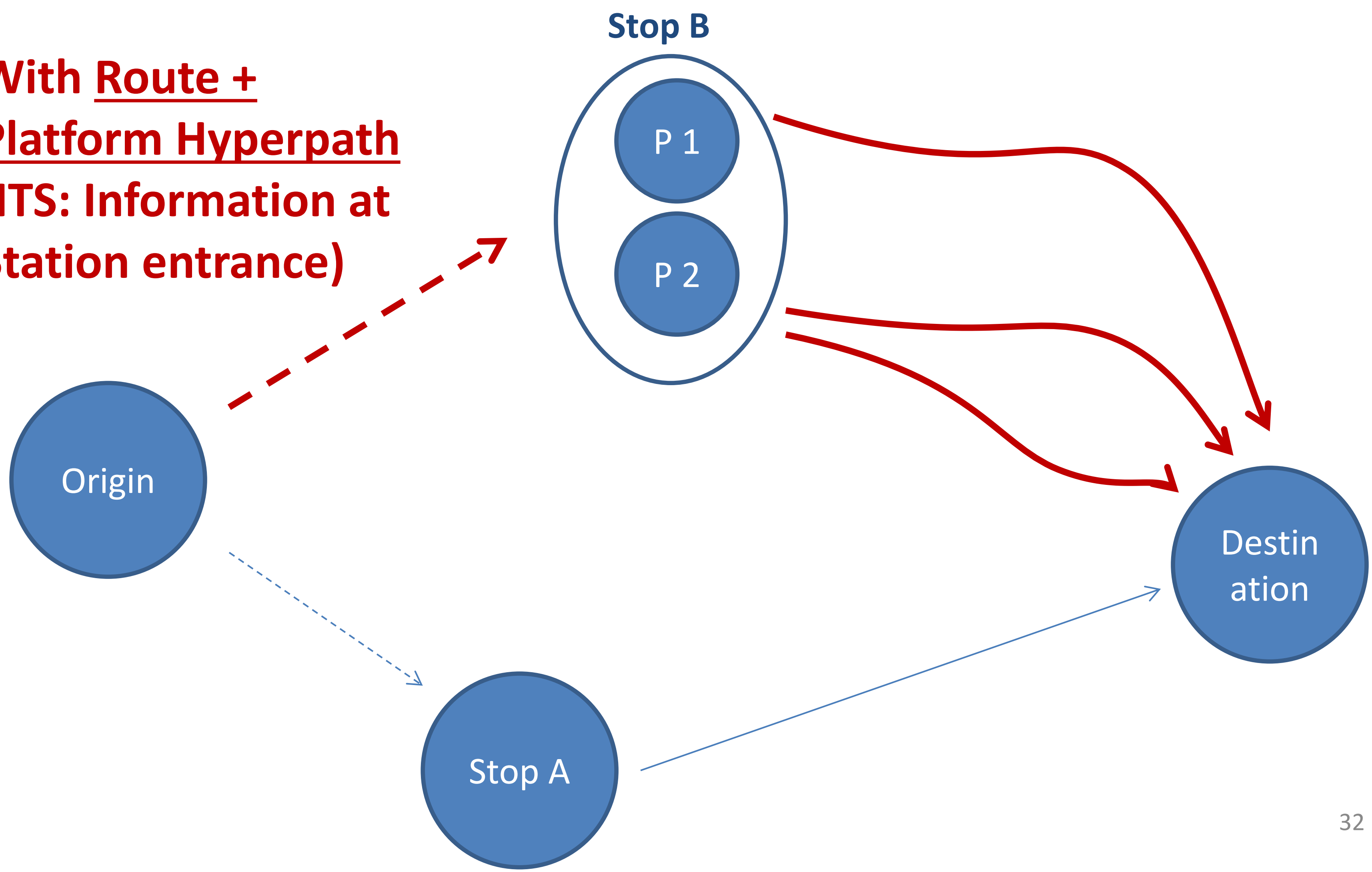
Network Example

With <u>platform hyperpath</u> consideration



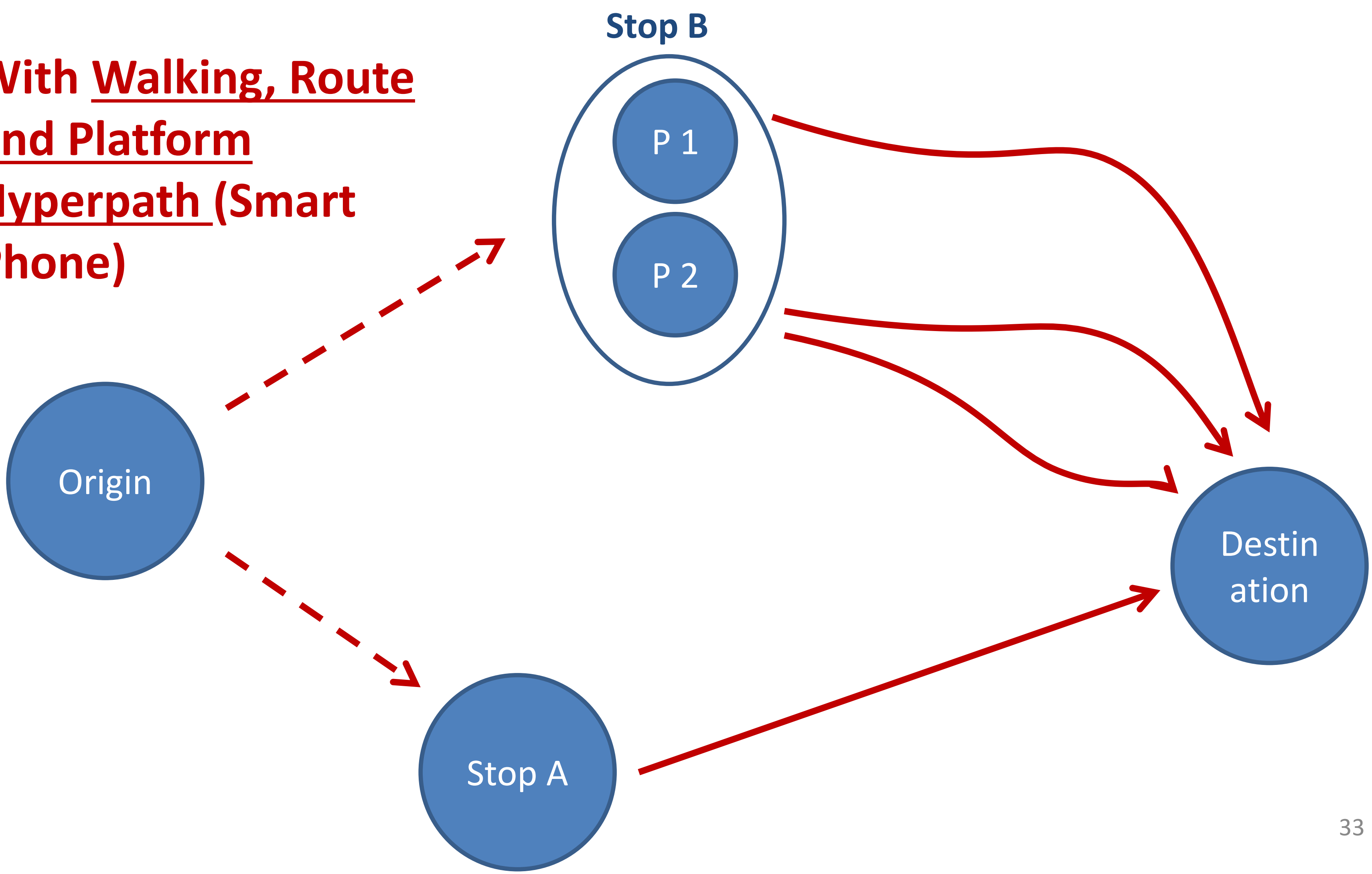
Network Example

With <u>Route + Platform Hyperpath</u> (ITS: Information at Station entrance)



Network Example

With <u>Walking, Route and Platform Hyperpath</u> (Smart Phone)



Note on Hyperpath vs Strategy

- Through reliable, “dynamic” information on service departures, route choice decision points move closer to passenger departure time: At decision point the hyperpath collapses into a single path.

- Therefore for the passenger the hyperpath might not necessarily become more complex, but only “from a modelling perspective”.

Overall Conclusions

- “Smart transit”, i.e. providing good information allows the traveller to benefit by less need to be risk-averse and by having wider options
- This might in turn help the system

- But there is also a danger that the traveller “outsmarts” the system
  - Headway perturbations
  - Focus on the shortest route
- …requiring possibly even more information and network management

Current work connected to the topic…

- Fare structures: Increasingly complex to manage and attract demand
  - Zones vs distance based and flat fares
  - Special discounts: OD pairs, peak times, loyalty rewards (per day, per month..), Shopping points
- Bus bunching and stop layout: in how far can "intelligent design" make a system robust against delays

Thank you

schmoecker@trans.kuciv.kyoto-u.ac.jp

References and acknowledgements to my co-authors

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