Stability analysis of Activity-Based models

Case study of the Tel Aviv Transportation Model

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Introduction

- Activity-Based Models (ABM) are disaggregate models that simulate individual decisions as random draw from choice sets, thus inserting random component to the results of model run.
- The Tel Aviv ABM structure is similar to other activity based models described in the literature.
- The model run is supposed to converge to the equilibrium between generated tours and corresponding level of service (LOS) data.

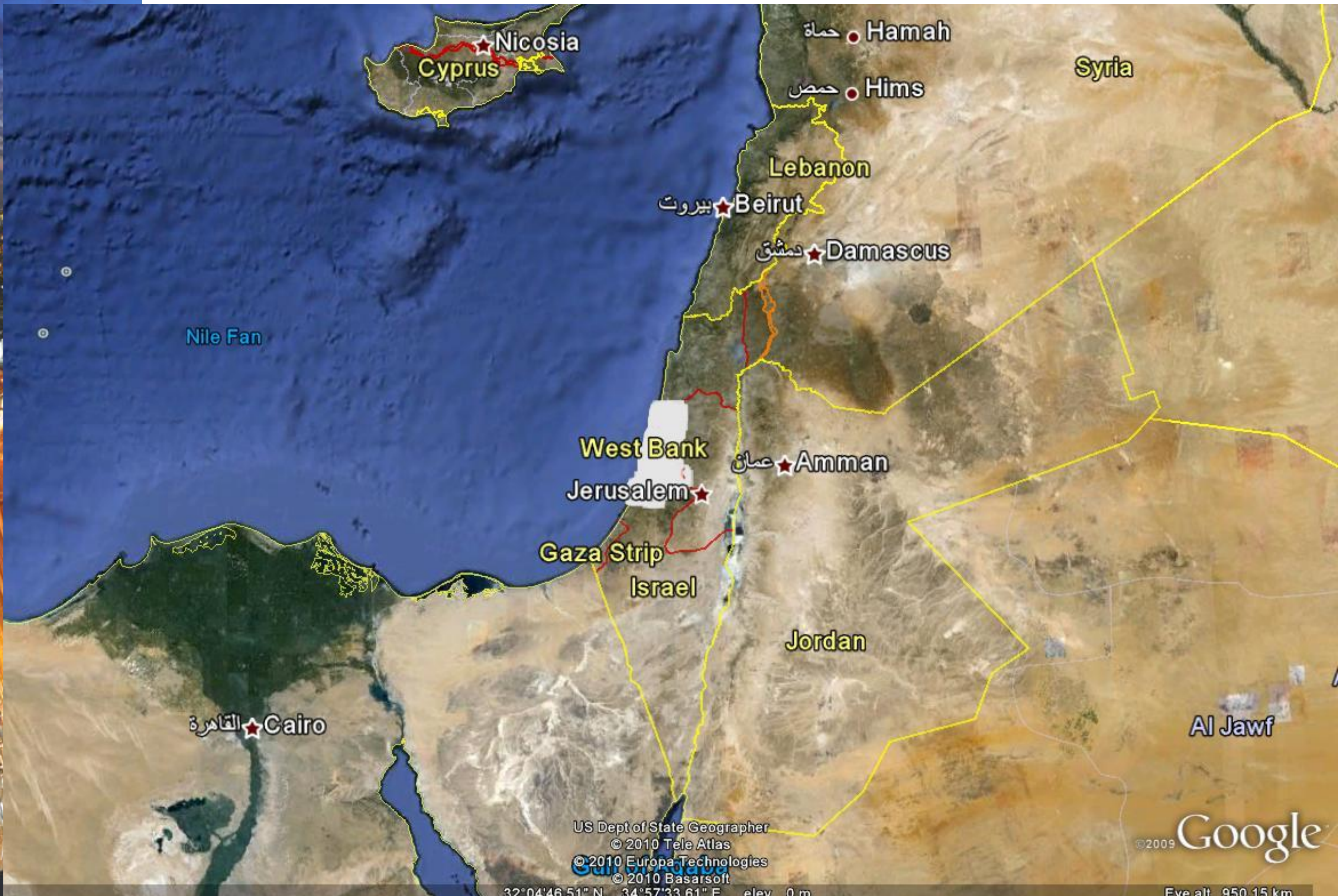
Goals of current study

- Experience of working with Tel Aviv ABM has revealed the need in conducting comprehensive analysis of the randomness of the model results, and in developing practical methods for the model stability monitoring and control.
- Therefore, the goals of the current study are:
  - Analyze sources of randomness and their influence on model results
  - Produce practical recommendations for correct use of the ABM

Literature Review

- Rasouli and Timmermans (2012) presented a review of uncertainty in travel demand forecasting models.
- Veldhuisen et al. (2000): effect of Monte Carlo draws on regional aggregate activity patterns
- Castiglione et al. (2003): variability of the forecasts due to random simulation error
- Bowman et al. (2006): techniques to establish convergence based on MSA
- Vovsha et al. (2008): discussion of practical ways to reach equilibrium within an activity-based model
- Cools et al. (2011): uncertainty related to the statistical distributions of random components

Tel Aviv Metropolitan Area



Tel Aviv Model

About 1,500 Km^2 and 3.3 million habitants in 2009

1,219 Traffic Analysis Zones

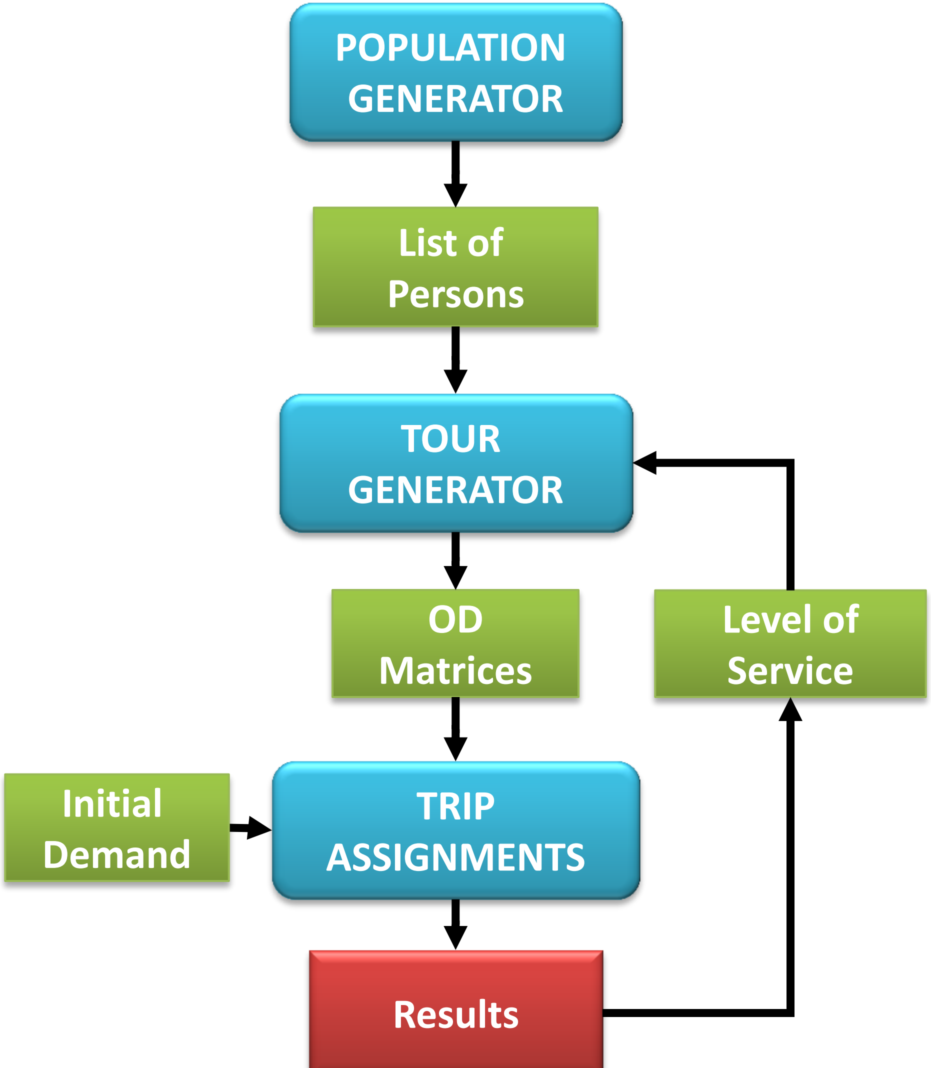
About 10,000 regular links

About 1,000 Transit lines

Main Modes: car driver, car passenger, taxi, bus, rail, Mass Transit (BRT/LRT)

Access modes: walk, transit, Park&Ride, Kiss&Ride

Tel Aviv Model (structure and implementation)



Implementation features

Disk space: 4 GB per project + 3GB common space

Memory required: 2GB

PG: stand alone application

TG: Special C# application

Assignments: Parallel work with 3 EMME banks on single PC

Run time: 40 min per iteration, 10% population sample, Intel i7 Quad processor

Model Structure

- The hierarchy of the models is fixed and the general model structure is similar to the Bowman and Ben-Akiva (2001) approach.
- The results from these models are translated into O-D travel matrices that are assigned to the network.
- For details on the model application, see Bekhor et al. (2011) and Shiftan et al. (2003).
- The following slide shows the main components of the Tour Generator.

TOUR GENERATOR

Main Activity

- Work
- Education
- Shopping
- Other
- No tour

Time Period (15 alternatives)

Destination of the Main Activity (1219 alternatives)

- Zone 1
- Zone 2
- Zone 3
- ...
- Zone 1219

Mode of the Main Activity (14 alternatives)

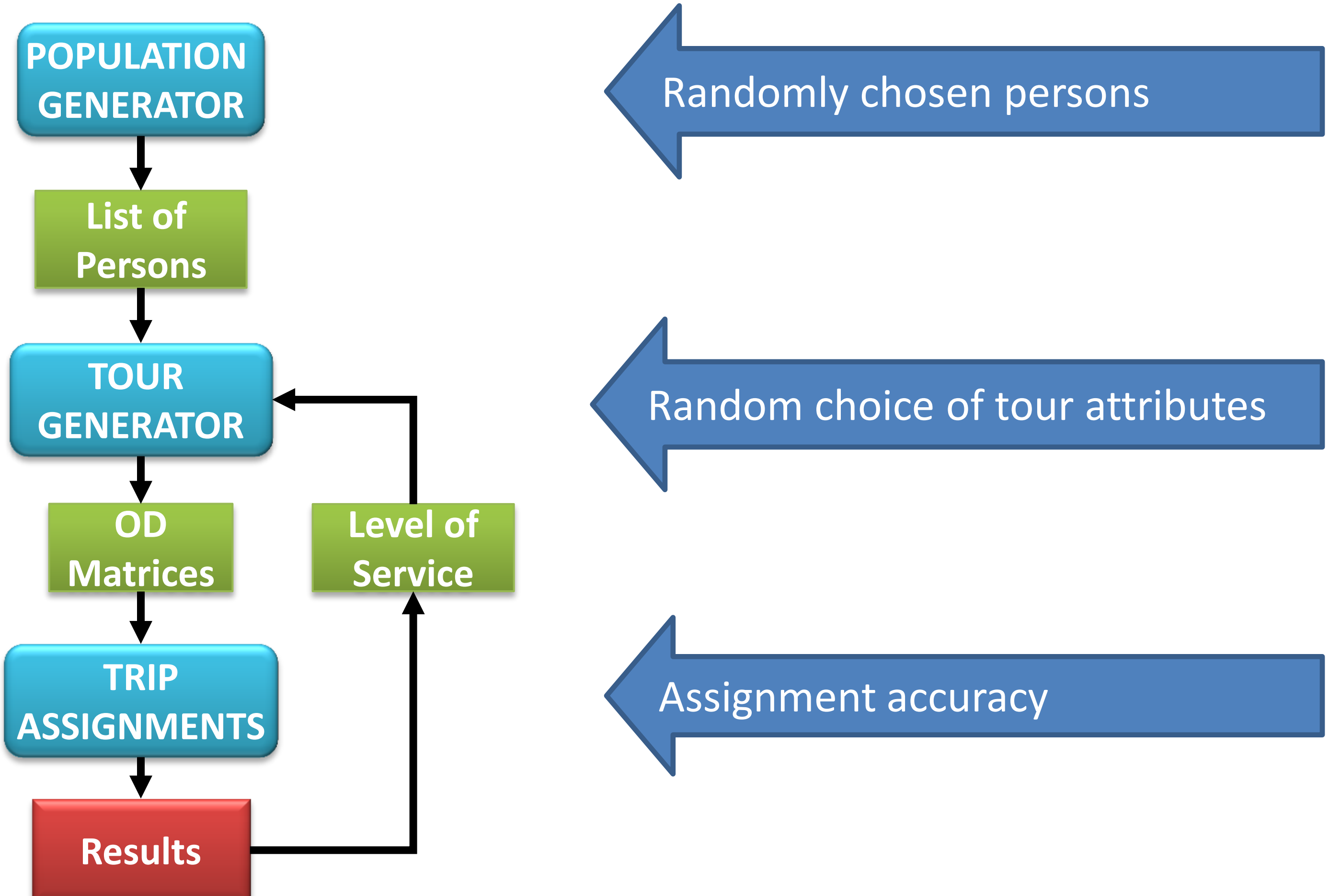
- Car Driver
- Car Passenger
- Taxi
- Bus
  - P&R, K&R, Walk
- Rail
- LRT / BRT
  - P&R, K&R, Walk, Bus

Intermediate Stops

- No stops
- Before the main activity
- After the main activity
- Before and After

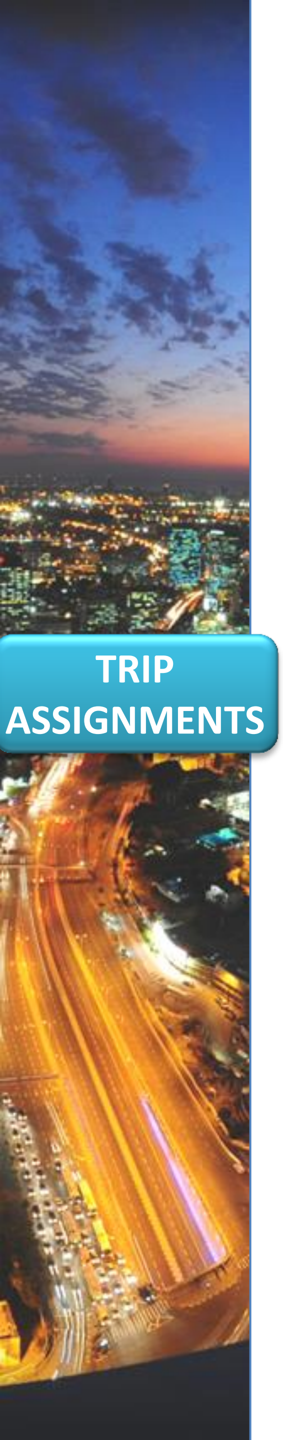
Errors/randomness sources

- There are three major model elements where the randomness or inaccuracy may occur:



Error sources to address

- Assignment accuracy — TRIP ASSIGNMENTS
- Tour Generator random component — TOUR GENERATOR
- Population sampling — POPULATION GENERATOR



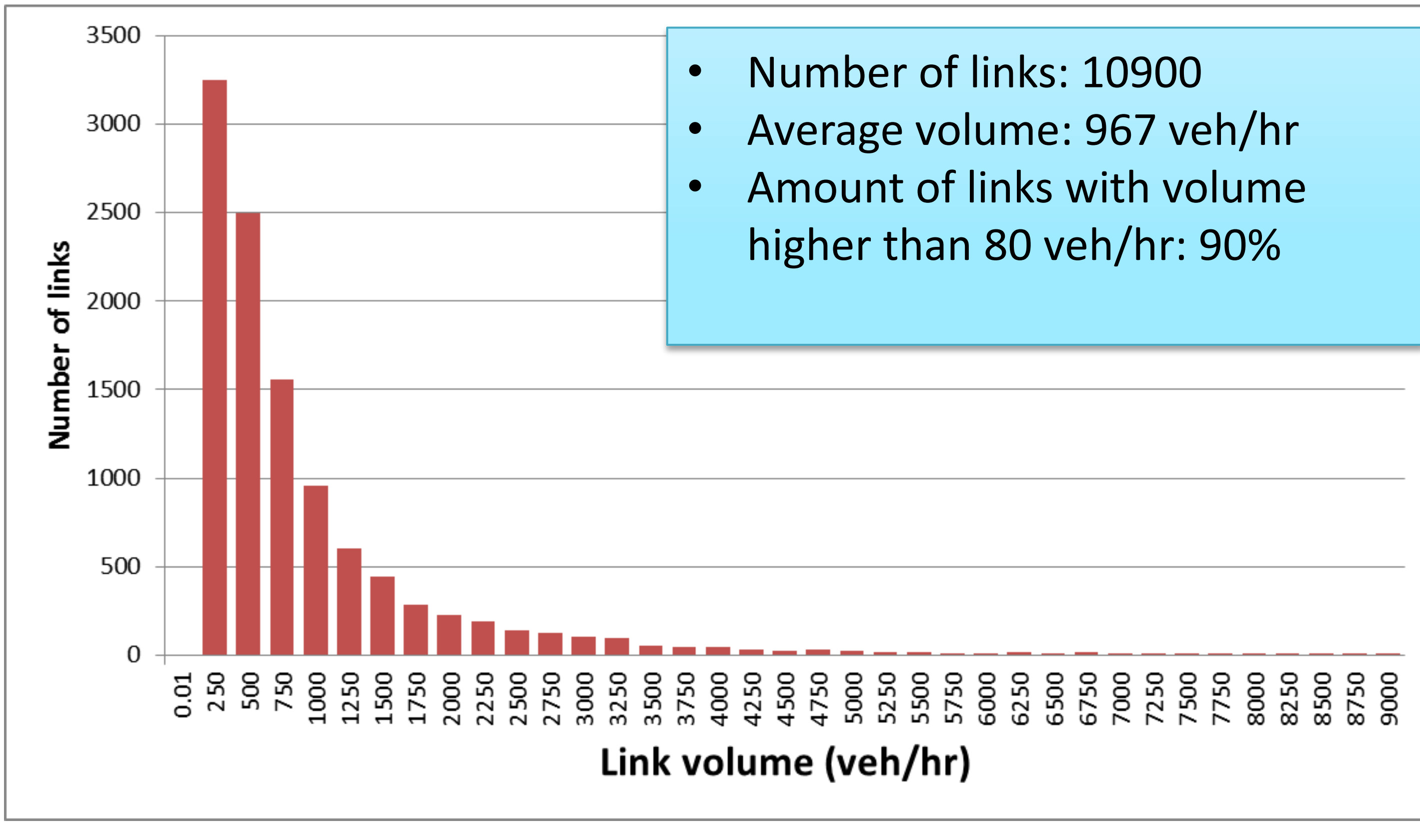
Assignment errors

- Three variants of traffic assignment implementation were compared:
  - Standard Frank-Wolfe algorithm (FW)
  - Frank-Wolfe algorithm with parallel computation (FWP)
  - Path-based traffic assignment (PBTA)

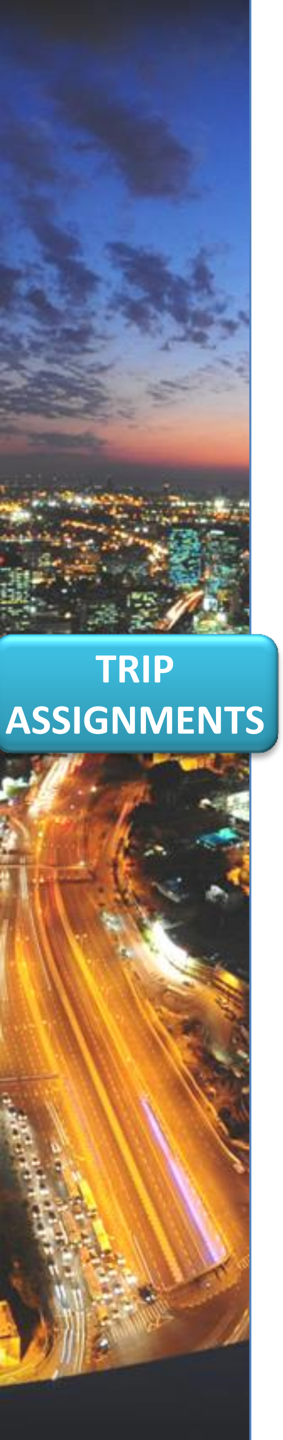
- The implementation accuracy was evaluated using distribution of errors in link segments.
- This measure is convenient for evaluation and comparison of errors of different nature.

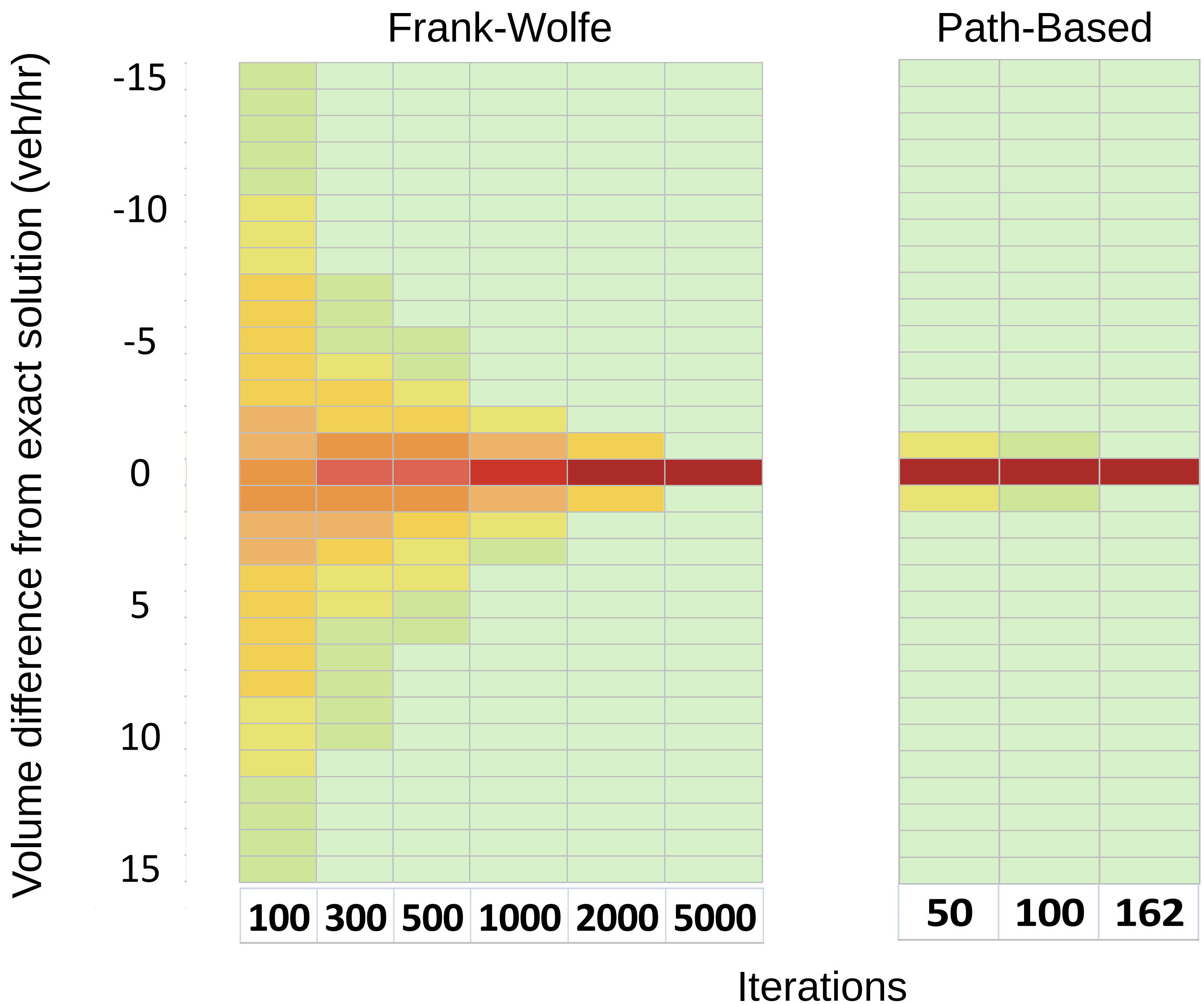
Assignment results: Distribution of link volumes

TRIP ASSIGNMENTS

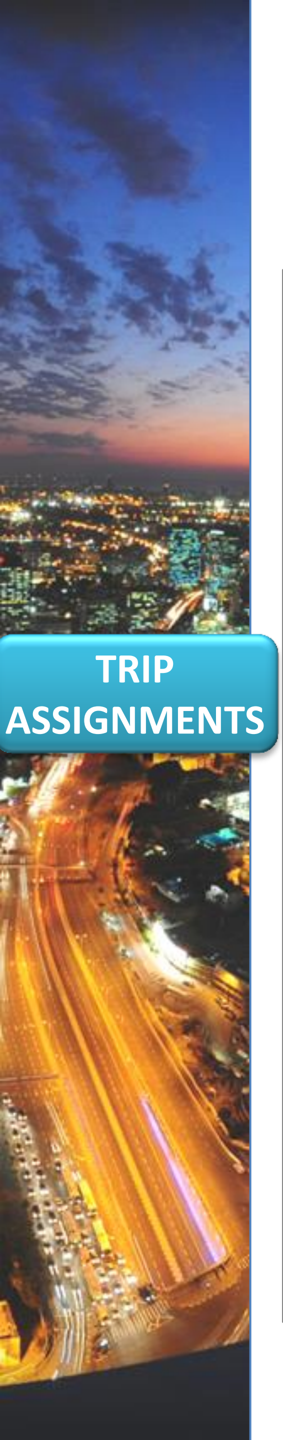


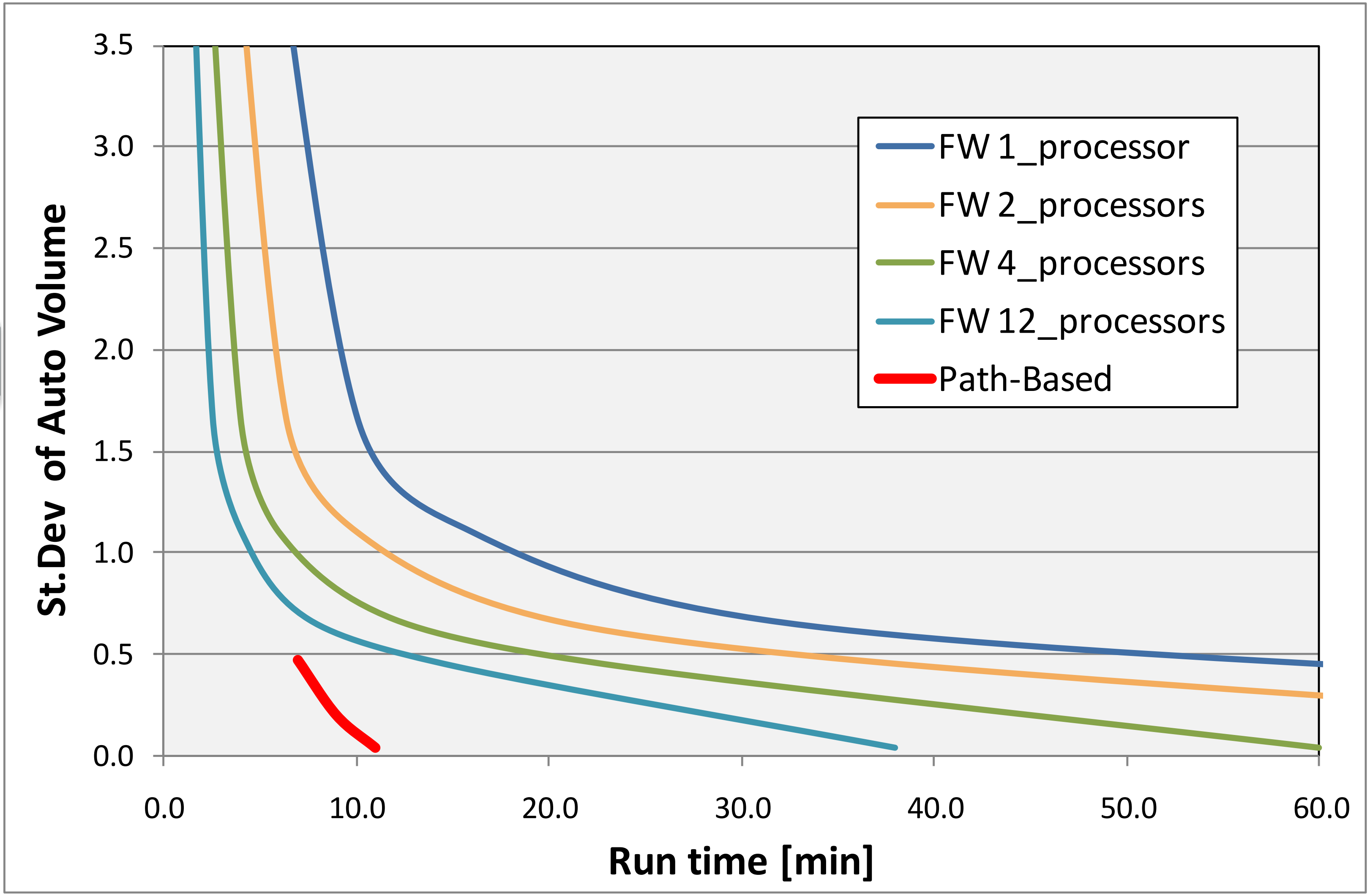
Assignment results: convergence





Assignment results: run times



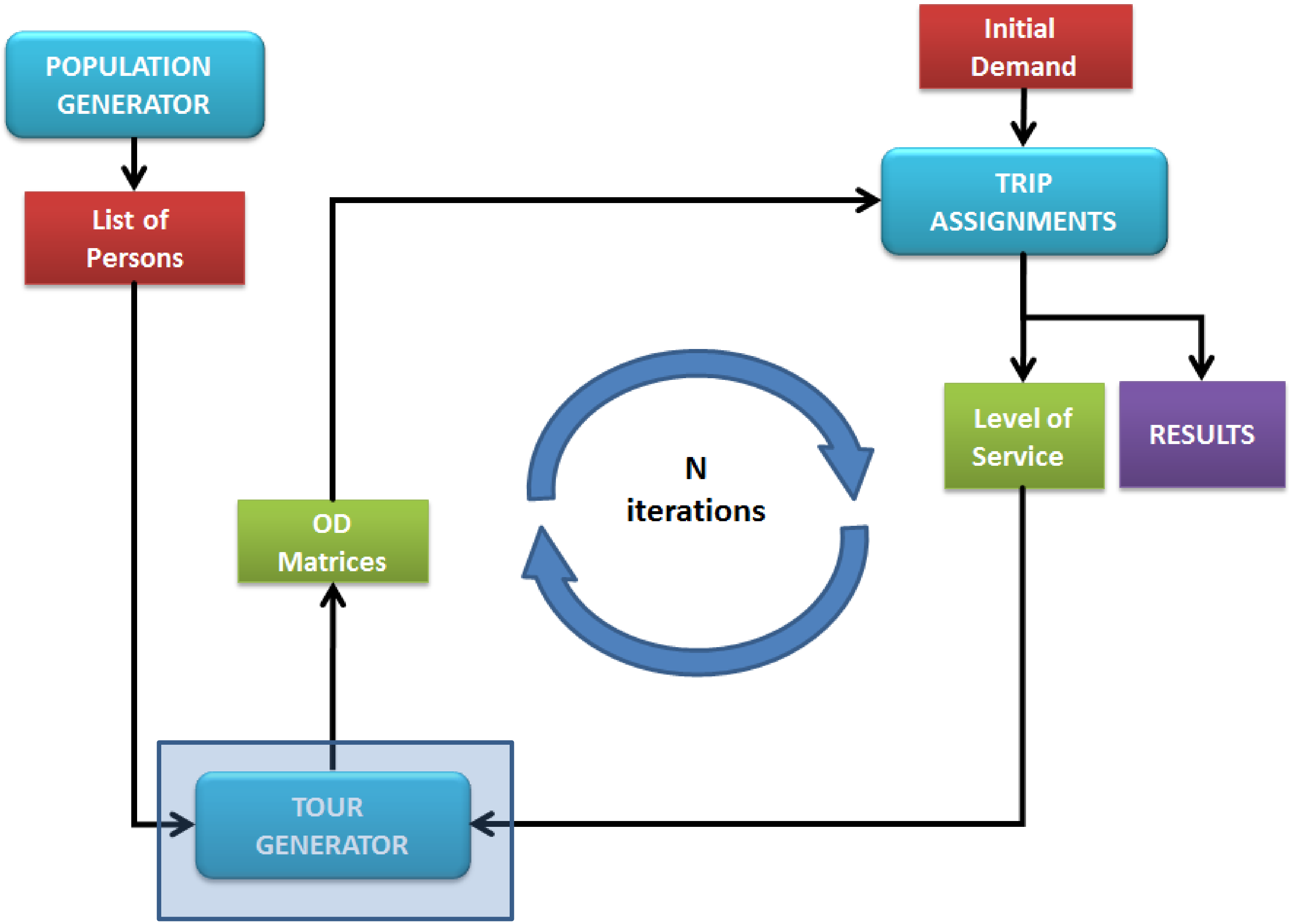


Assignment errors: conclusions

- The analysis of the assignment implementations has shown that usage of **path-based algorithm** may practically eliminate the assignment error.
- The time savings of the PB algorithm is further expanded due to extensive use of path analysis allowing to obtain various characteristics of assignment results very fast.
- The FW and FWP algorithms require conducting additional time consuming assignments.

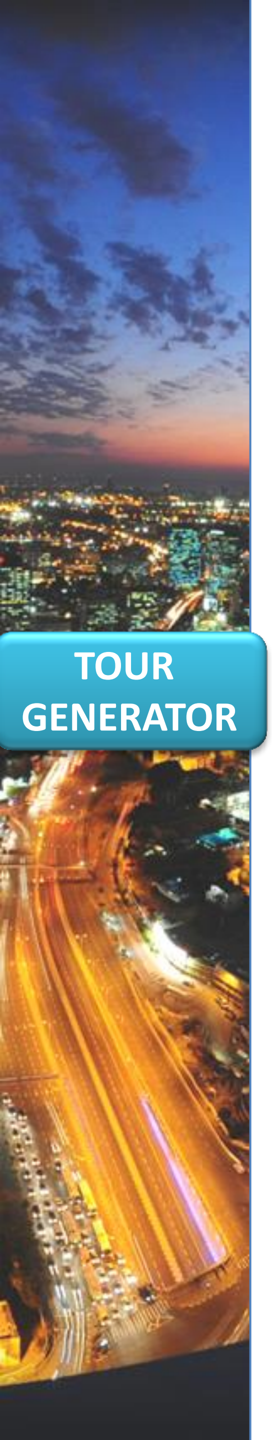
TOUR GENERATOR

Tour Generator random component



Tour Generator - Simulation error

- The starting point of the analysis is the demand matrices that result from the TG component.
- According to the flowchart presented in the previous slide, the individual random choices are aggregated to form the demand matrices.
- There are over 30 demand matrices generated by the model for different modes and periods of day.
- To analyse the TG randomness effects, we consider the car demand matrix for the AM period.



The demand matrices are random

- The random draw of choices for each person in a sample (Activity, Destination, Period of Day, Mode, etc.) results in random matrices, for example:

Car demand, Iteration *n*

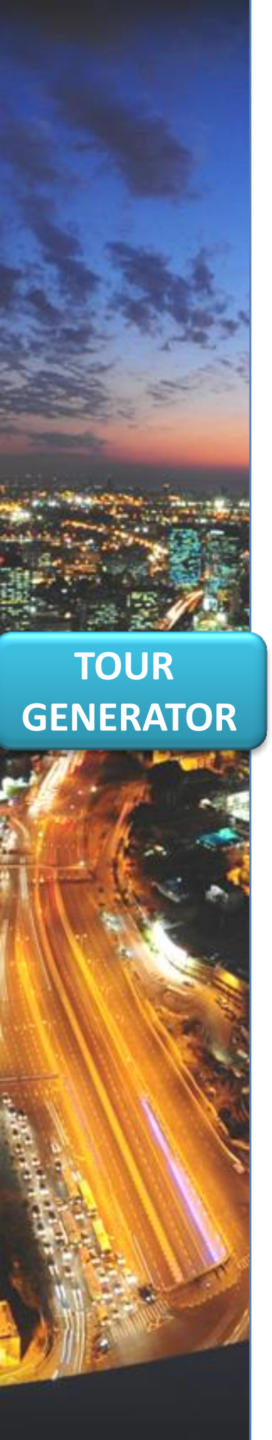
		Destination						
		1506	1509	1601	1602	1605	1903	1921
Origin	1105	10	10	10	10	20	0	20
	1106	0	0	0	0	10	0	0
	1107	0	0	10	0	0	0	20
	1108	0	10	10	10	20	0	0
	1109	0	0	20	0	20	0	0

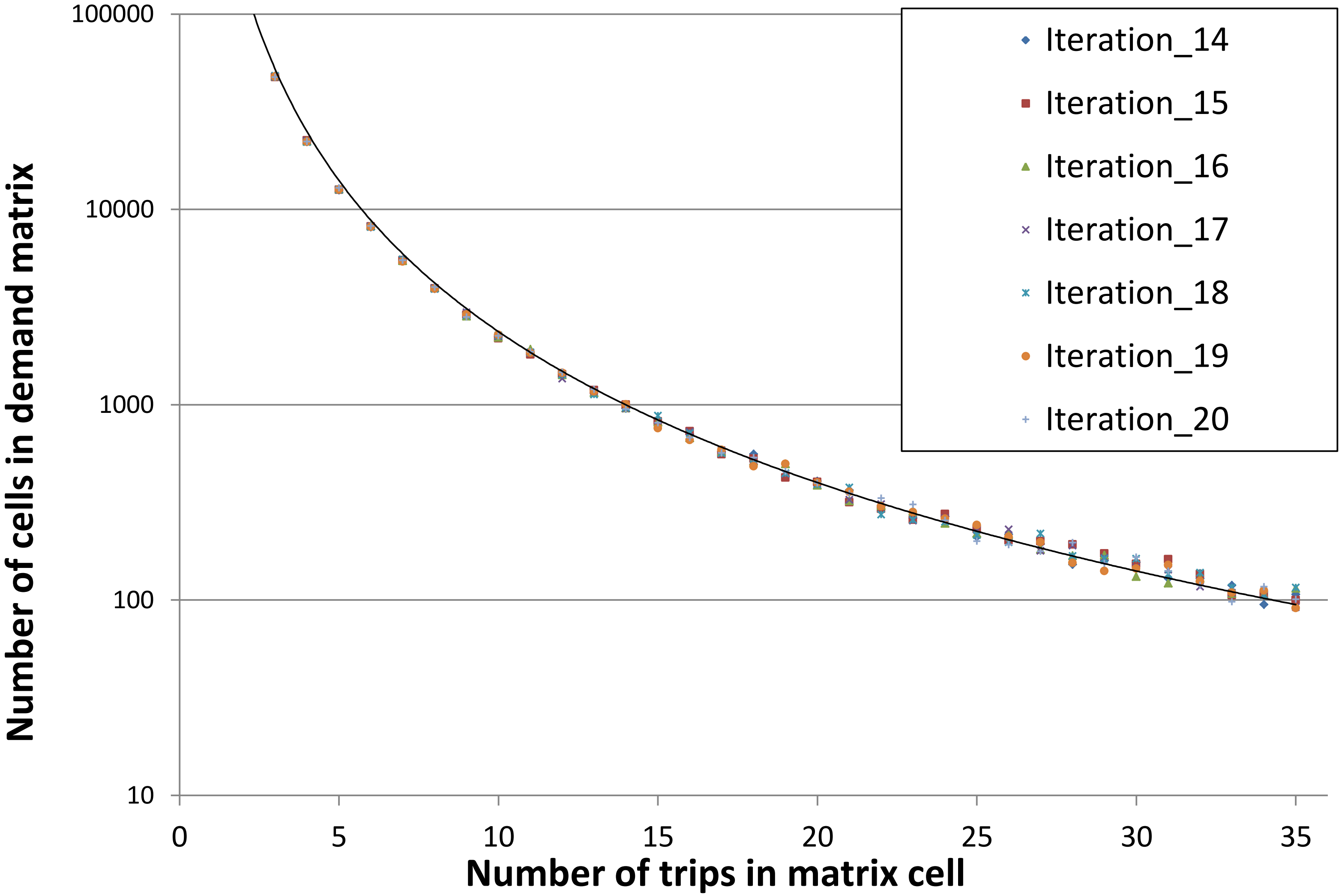
Car demand, Iteration *n*+1

		Destination						
		1506	1509	1601	1602	1605	1903	1921
Origin	1105	0	0	0	0	10	10	10
	1106	0	0	10	0	30	0	0
	1107	0	0	0	0	10	10	10
	1108	0	0	0	10	10	20	10
	1109	10	10	0	0	10	0	0

- Although the overall distribution of trips in the matrix is quite stable, there are considerable changes at the cell level.

Distribution of trips in the car demand matrix for different iterations



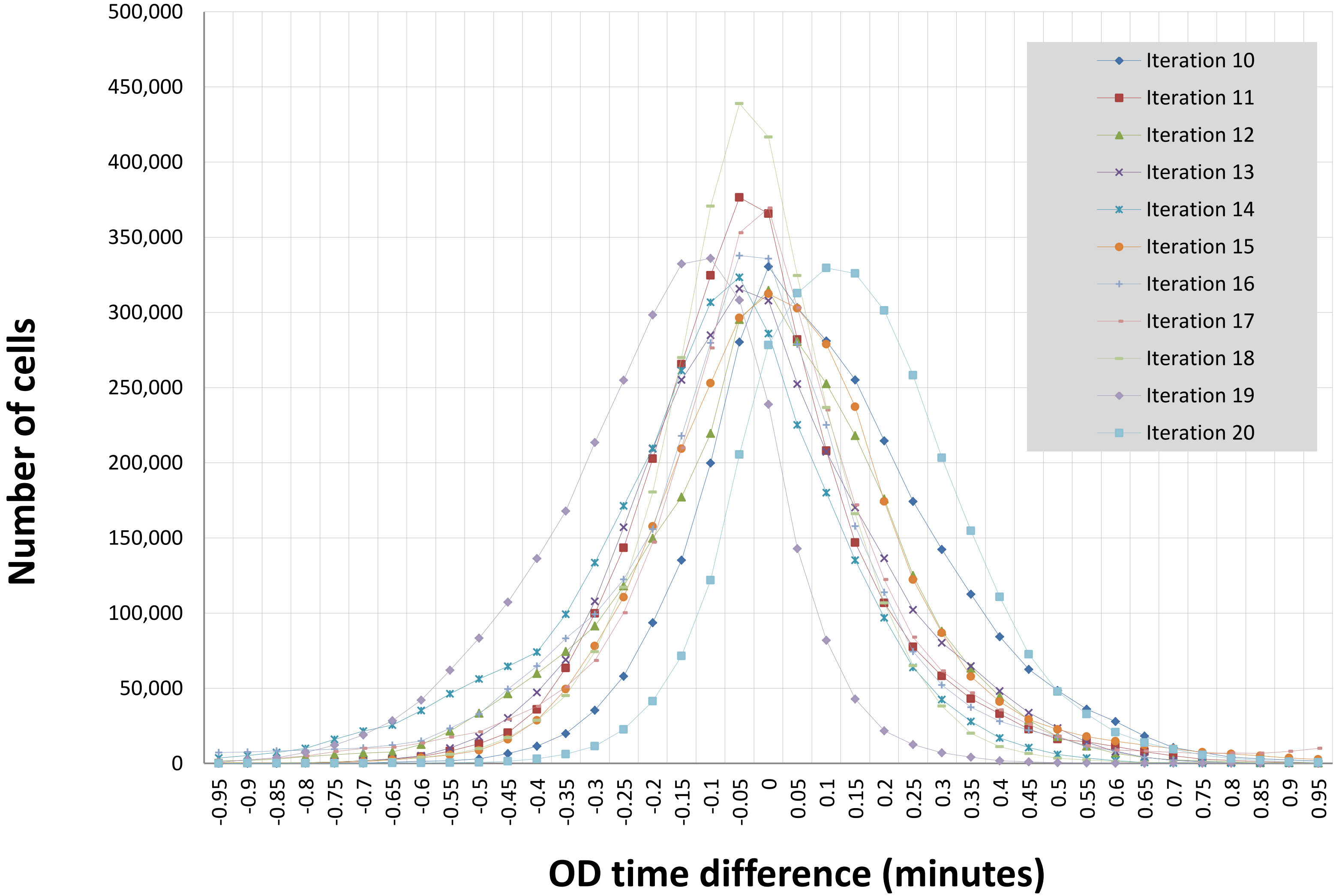


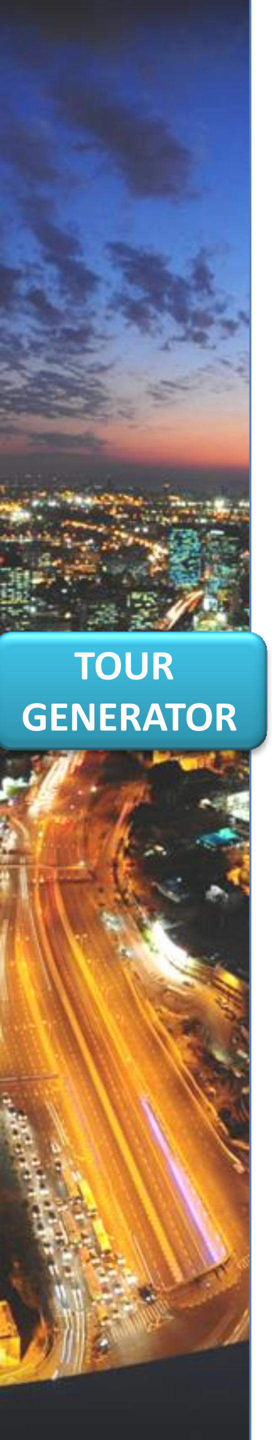
Sparseness of demand matrices

Parameter	Average	Standard Deviation
Total matrix cells (1219 * 1219)	1,485,961 (100%)	
Total non-empty matrix cells	270,101 (18%)	403
Total matrix cells with one trip	150,362 (10%)	382
Total non-empty cells in iteration (n), corresponding to empty cells in iteration (n+1)	117,068 (8%)	364
Total cells with one trip in iteration (n), corresponding to empty cells in iteration (n+1)	93,152 (6%)	346

Distribution of OD time differences

TOUR GENERATOR

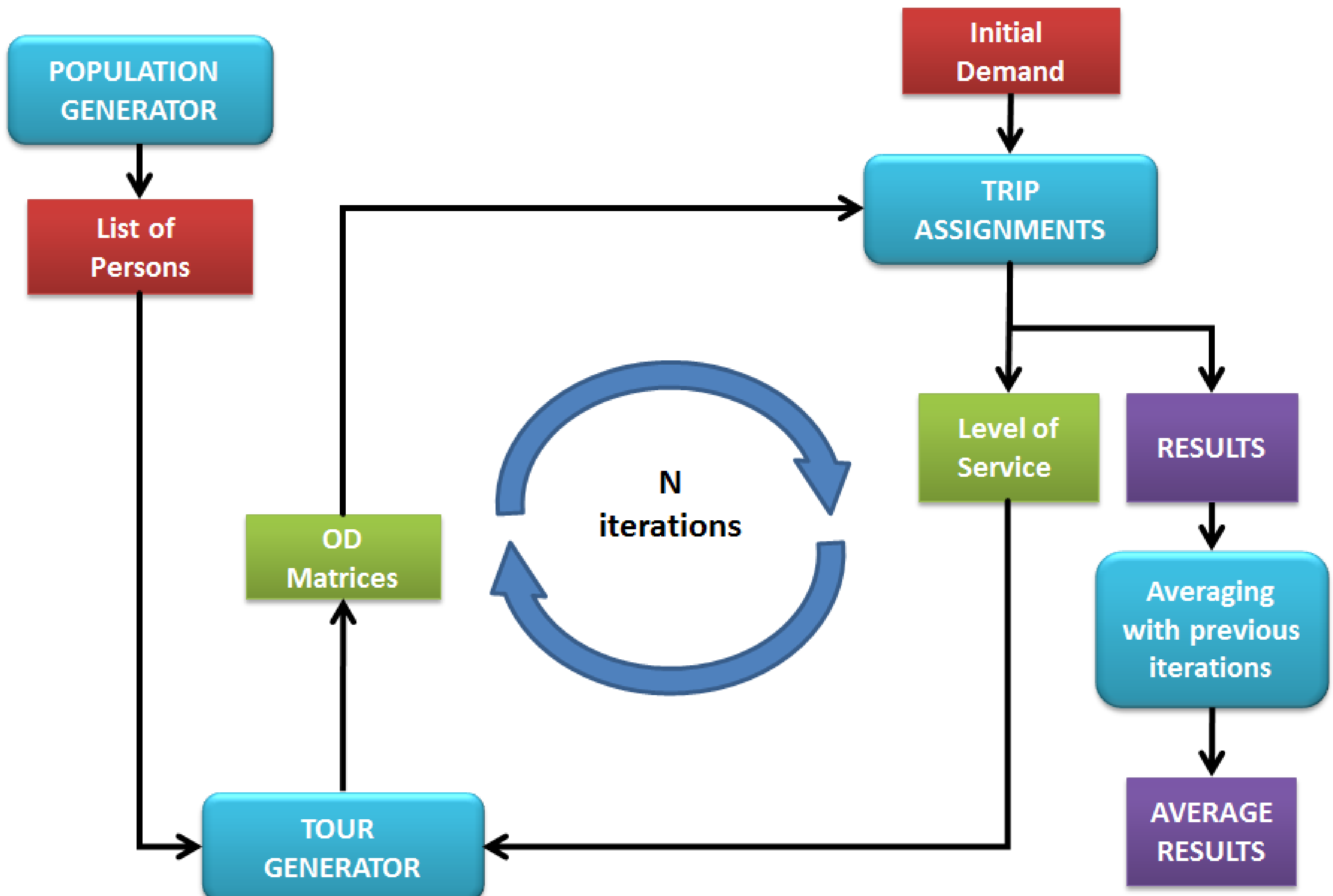




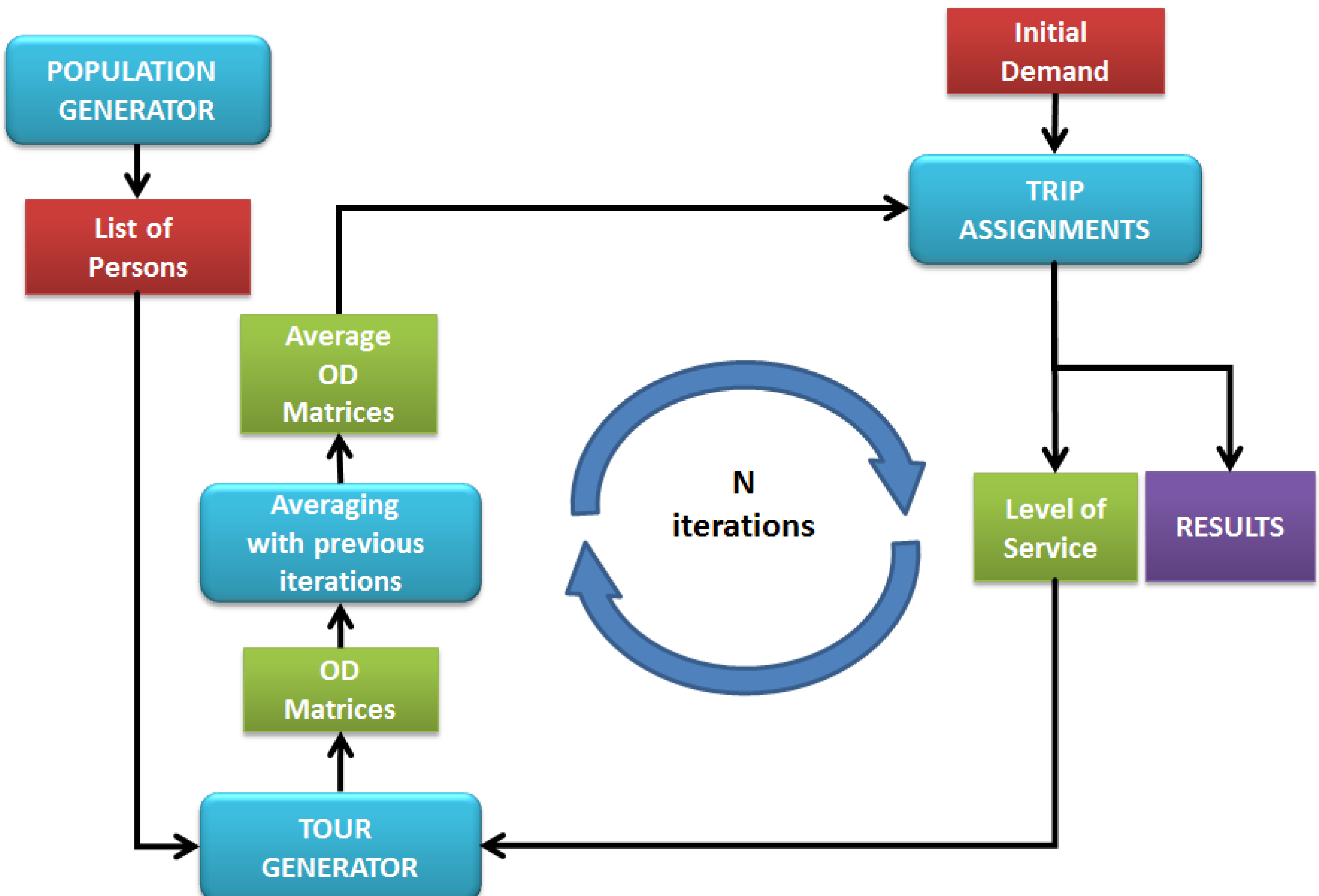
Averaging methods

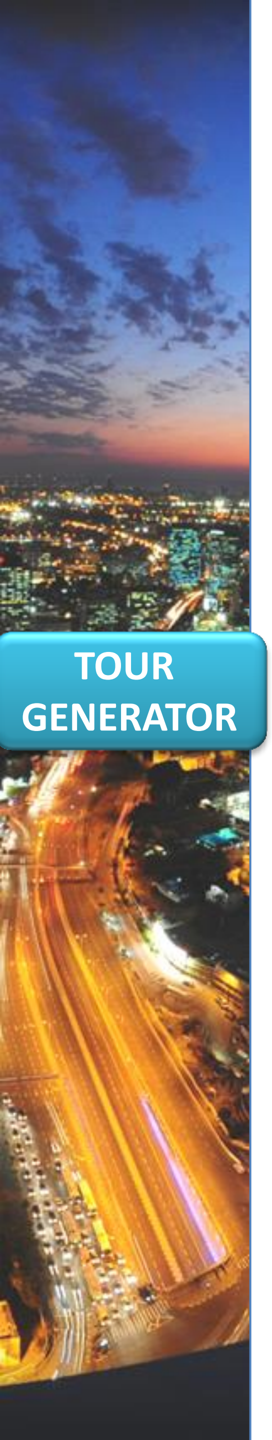
- Different averaging methods can be used to stabilize the model run results.
- We consider 3 averaging methods, which are respectively presented in the following slides:
- "MSA-R":
  - $\widehat{VHT}_a^{(n+1)} = \frac{\widehat{VHT}_a^{(n)}}{n+1} + \left(1 - \frac{1}{n+1}\right) VHT_a^{(n+1)}$
- "MSA-M":
  - $\hat{T}_{OD}^{(n+1)} = \frac{\hat{T}_{OD}^{(n)}}{n+1} + \left(1 - \frac{1}{n+1}\right) T_{OD}^{(n+1)}$
- "Quasi-Aggregation":
  - $\hat{T}_{OD}^{(n+1)} = \frac{\hat{T}_{OD}^{(n)}}{n+1} + \left(1 - \frac{1}{n+1}\right) \frac{1}{K} \sum_{k=1}^{K} T_{OD}^{(n+1,k)}$

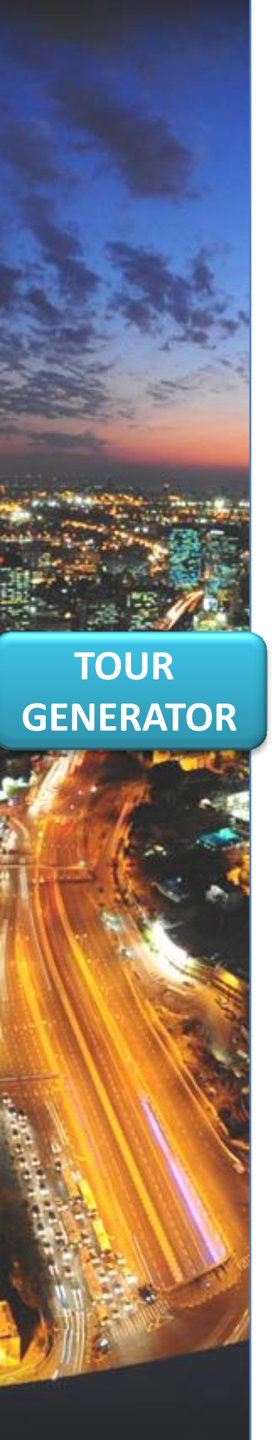
MSA-R method


POPULATION GENERATOR
List of Persons
TOUR GENERATOR
OD Matrices
N iterations
Initial Demand
TRIP ASSIGNMENTS
Level of Service
RESULTS
Averaging with previous iterations
AVERAGE RESULTS

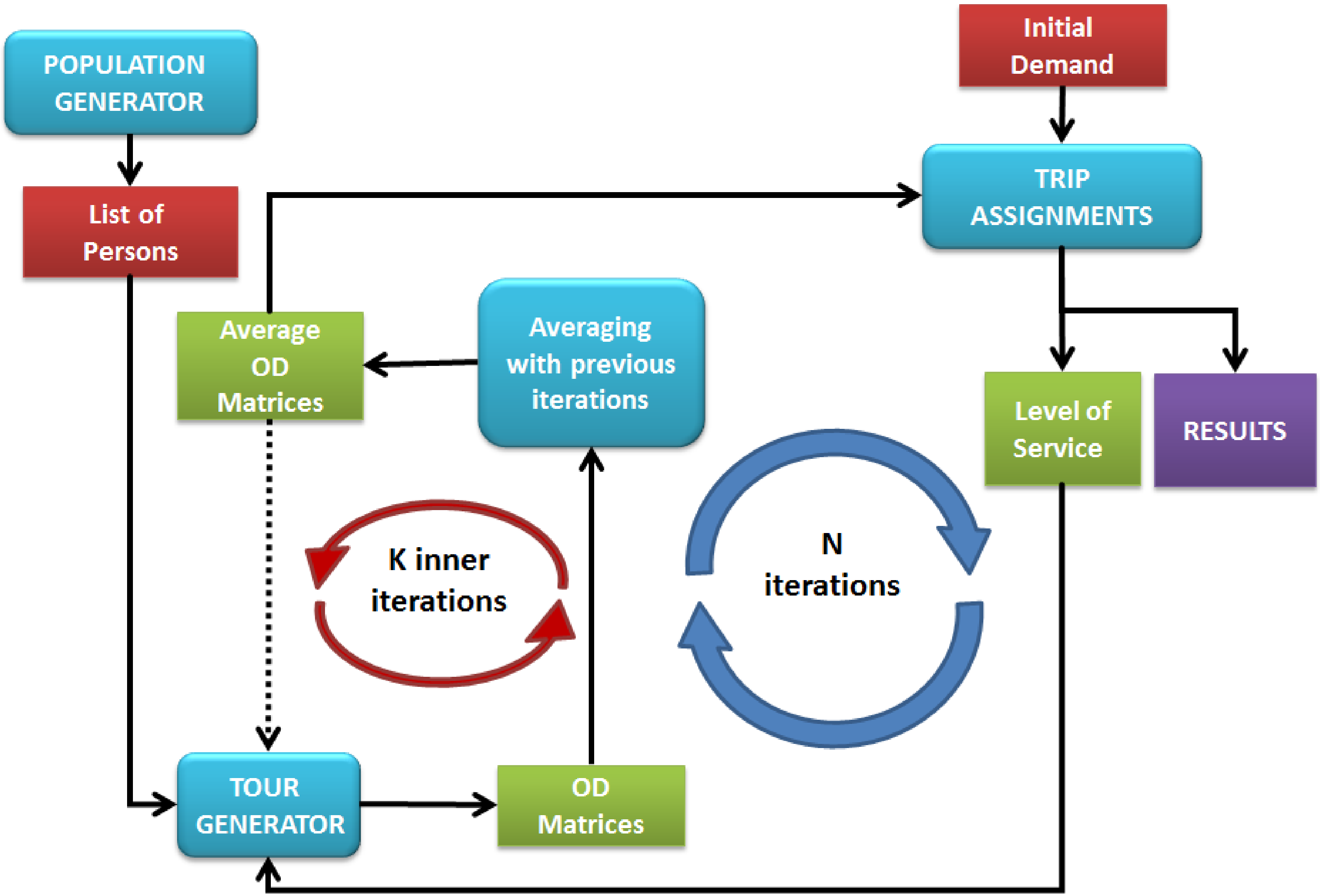
MSA-M method

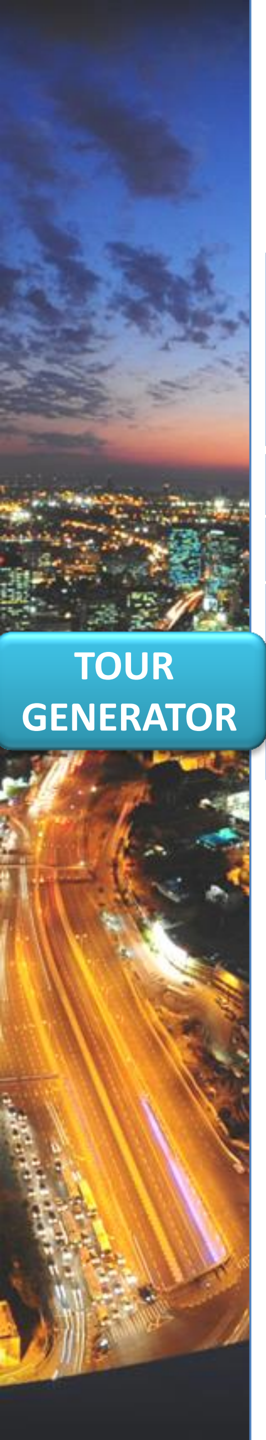

POPULATION GENERATOR
List of Persons
Initial Demand
TRIP ASSIGNMENTS
Average OD Matrices
Averaging with previous iterations
OD Matrices
TOUR GENERATOR
N iterations
Level of Service
RESULTS


TOUR GENERATOR



"Quasi-aggregation" method



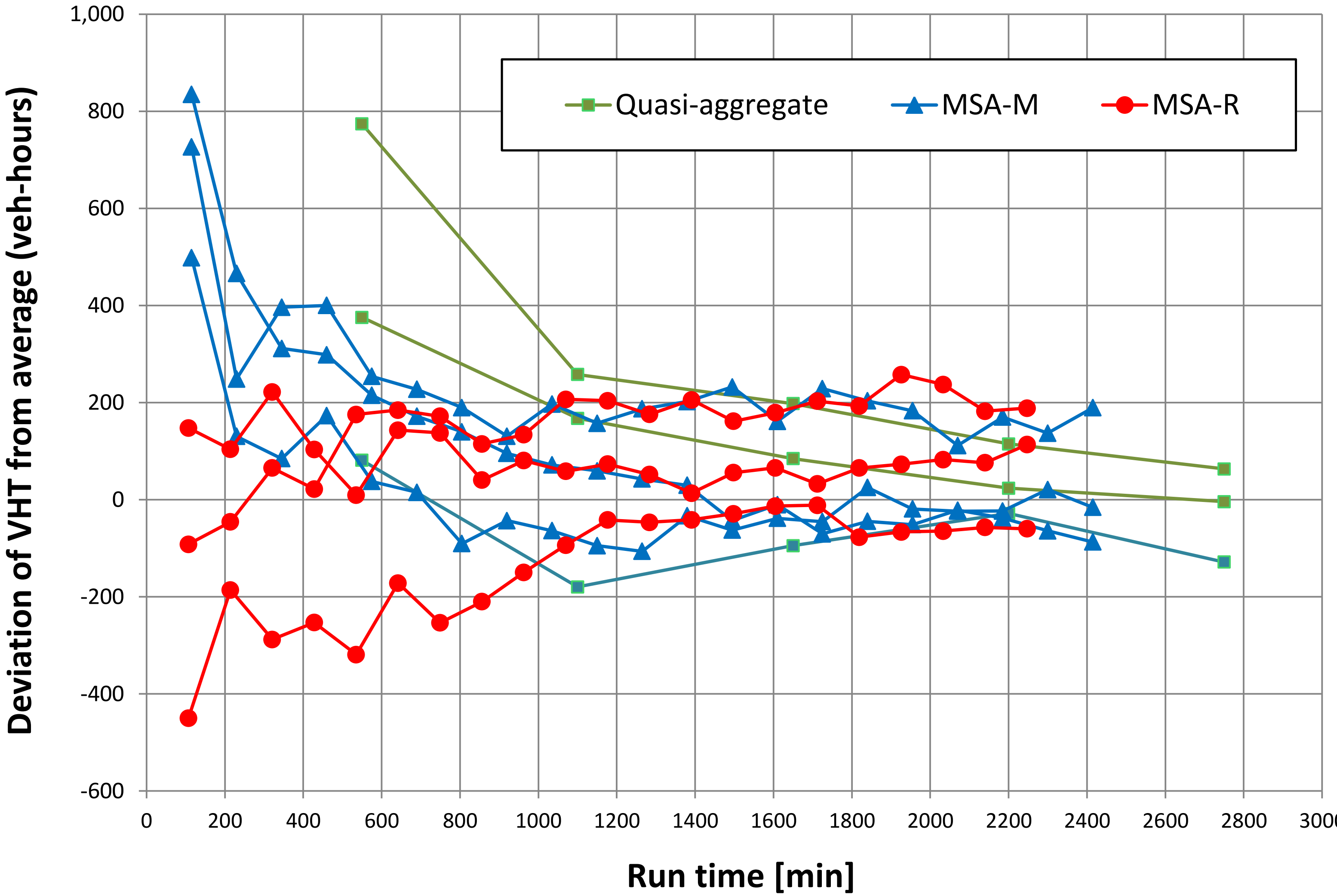


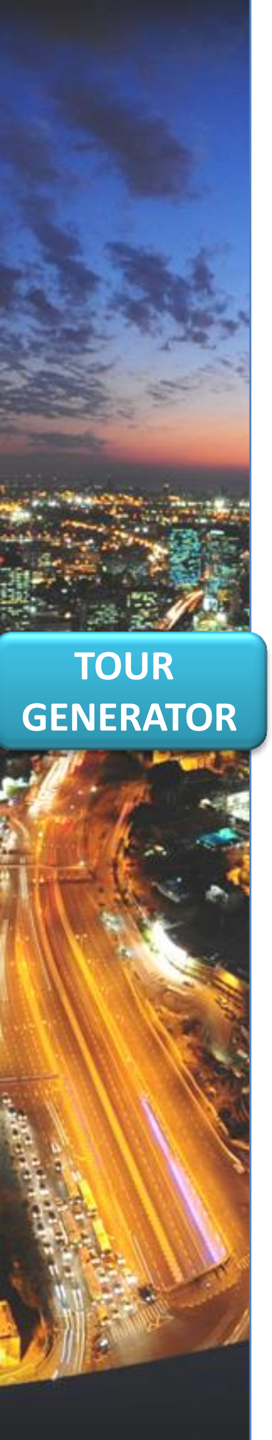
Convergence of averaging procedures

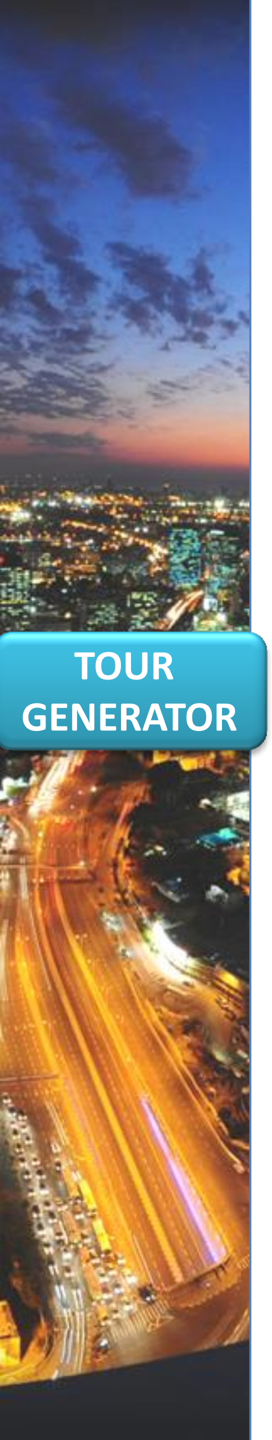
Averaging procedure	Number of iterations	Standard Deviation of VHT at last iteration (computed for 3 runs)	Average deviation of last iteration VHT from global average
MSA-R	20	120	-33
MSA-M	20	140	-9
Quasi-Aggregation	16 (6 inner each)	35	43

- The convergence rate of all procedures follows the "$\sqrt{n}$" rule of thumb:
- For MSA-R and MSA-M the VHT standard deviation decreases from the original 420 to 120-140 after 20 iterations, that is comparable to the expected $420/\sqrt{20} = 93.9$;
- The resulting VHT standard deviation of 35 for Quasi-aggregate procedure after 16 iterations with 6 inner iterations each is even closer to $420\,/\sqrt{16 \times 6} = 42.8$.

Convergence of averaging procedures as a function of run time







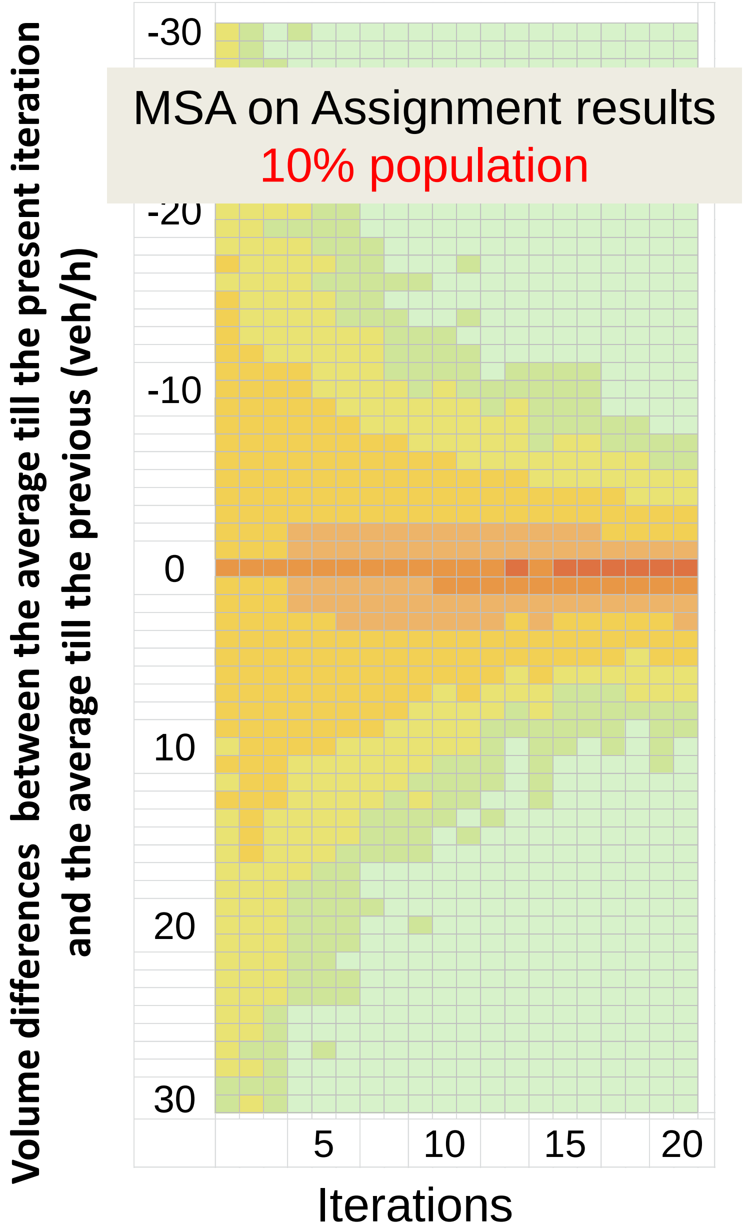
Tour Generator (conclusions)

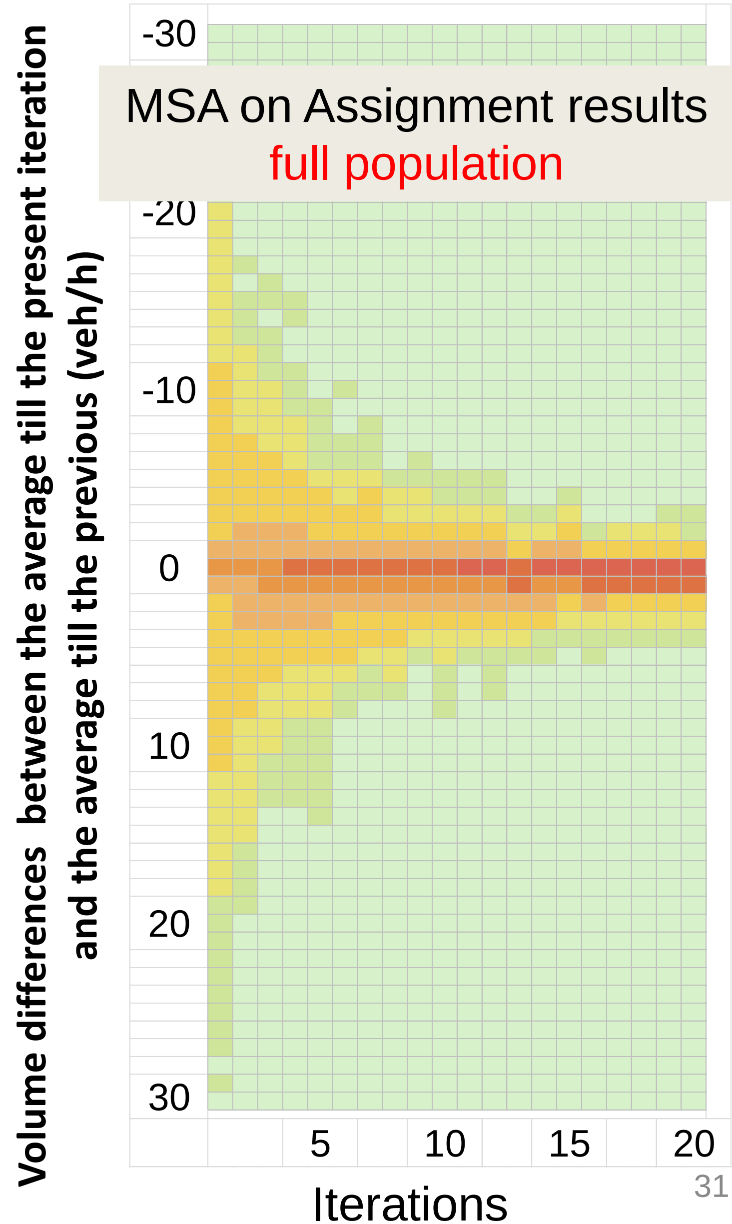
- All three arrangements of model run with averaging results converge similarly.
- Note that the MSA-M procedure requires more time for assignments than MSA-R, since for path-based assignment used the run time depends on number of non-zero cells in demand matrix, and in MSA-M procedure the number of such cells increases with the iterations, whereas in MSA-R this number is almost constant.
- Further, Quasi-aggregate procedure has different proportion between number of TG runs and number of assignment runs, depending on amount of inner iterations.

Population Generator random component

- PG generates list of individuals with random characteristics based on forecast of aggregate control variables.
- In addition, in order to accelerate the model's run, a sample from the full population is often used.
  - In this case, a sample is taken randomly, and trips of each person in a sample take proper weight to assure correct total number of trips in a system.
- In this presentation only errors related to the population sampling will be addressed.

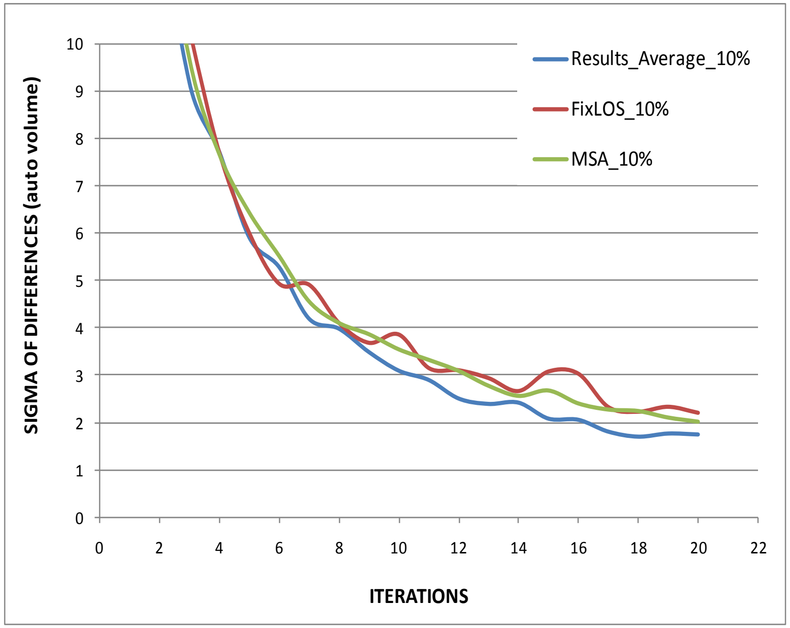
Effect of Sampling: Model Convergence



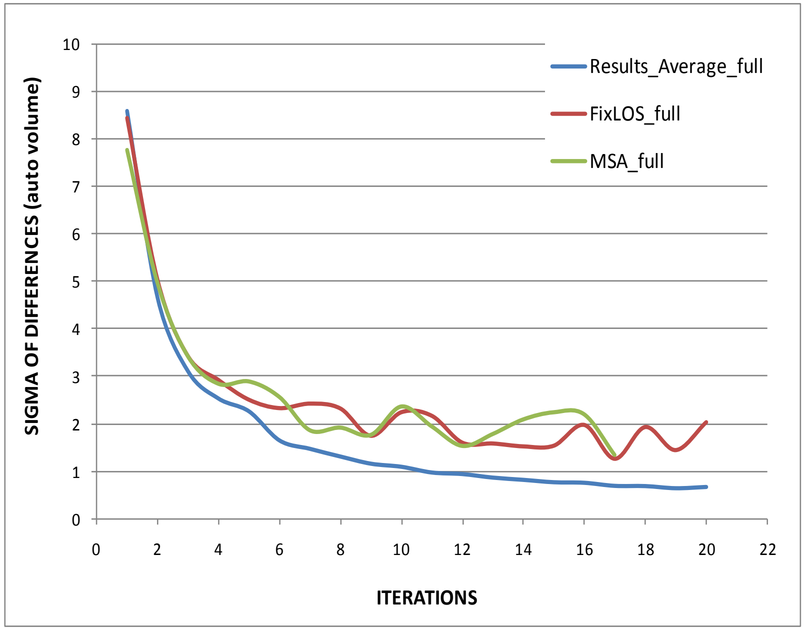


Effect of Sampling: Model Convergence

10% of Population



Full Population

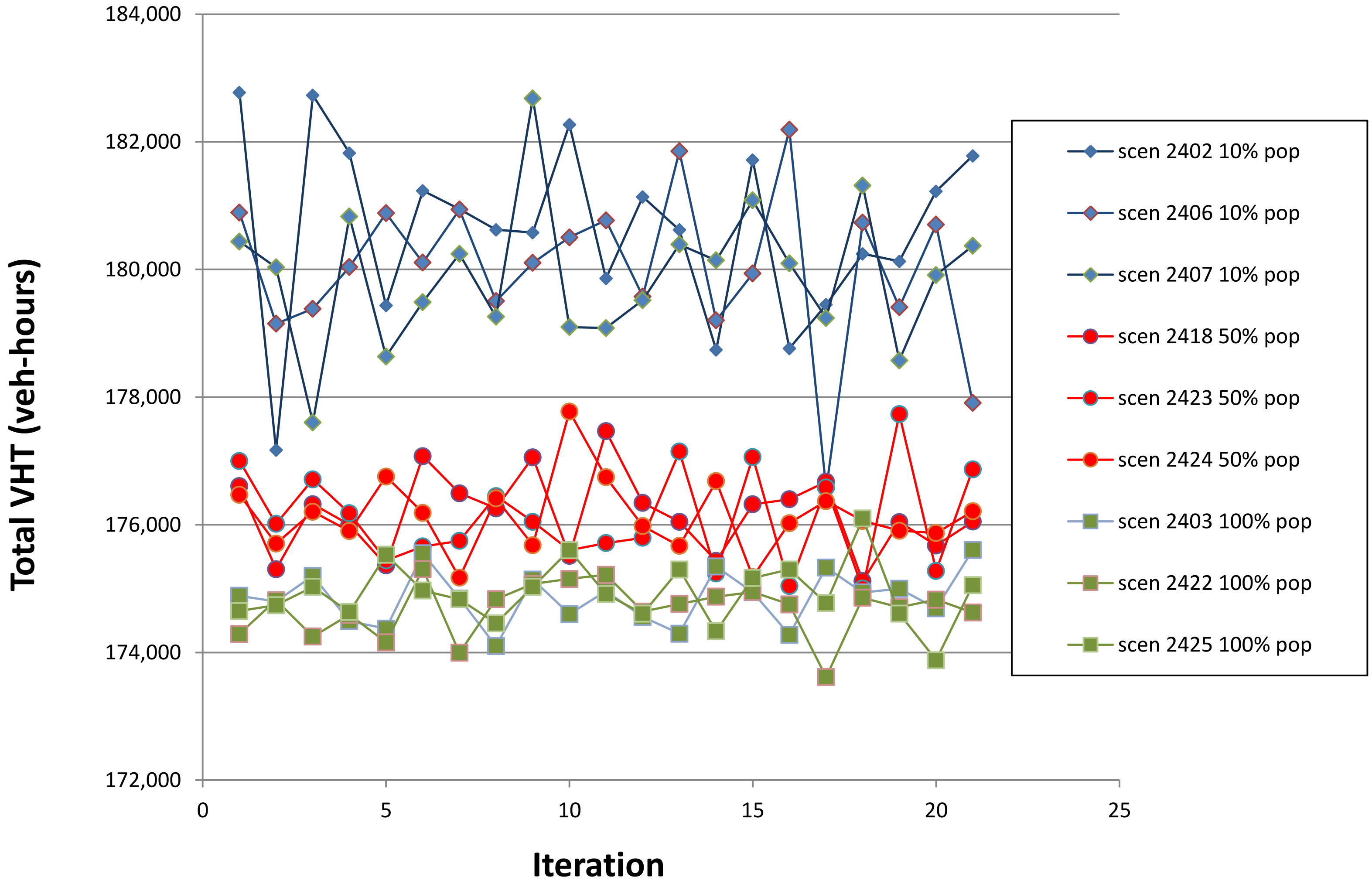


Convergence of ABM for different sample sizes

Sample size s	Average VHT	Standard deviation of VHT variations		Standard deviation after 20 iterations	
		Observed	Theoretical $\sigma_{100\%}/\sqrt{s}$	Observed	Theoretical $\sigma/\sqrt{20}$
100%	174,700	420	420	120	92.9
50%	176,150	650	594	160	145.3
10%	180,320	1,250	1,330	250	279.5

The results indicate that the "$\sqrt{n}$" rule of thumb works both for the number of iterations and for the sampling rate.

Variations of VHT for different population sample size



Population Generator - conclusions

- The sample size affects the stationary point of ABM results.
- In addition, population sampling does not bring any significant savings in number of iterations, if the goal is to assure certain accuracy of ABM results.
- This is because the sampling would require more iterations to converge to the same accuracy in comparison to the full sample.

Conclusions

- Three sources of the ABM results instability were analyzed: random population sampling, random tour generation, and assignment procedures.
- In line with previous studies, the effect of assignment procedures may be practically eliminated when using path based assignment algorithms.
- The effect of randomness of tour generation may be decreased significantly by averaging the results of model run.
- The analysis of ABM stability allows developing practical measures for performing estimation of transportation projects with controlled accuracy

Further research

Population sampling increases the efficiency of the model run, but the relationship between model steady states with different samples required further analysis.

Two major issues emerged from the presented work:

- Analysis of the ABM steady states
- More profound study of errors related to Population generator: uncertainty of **synthetic population** created from limited set of aggregate control variables