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Optimal Control Strategy for Relief Supply Behavior after a Major Disaster

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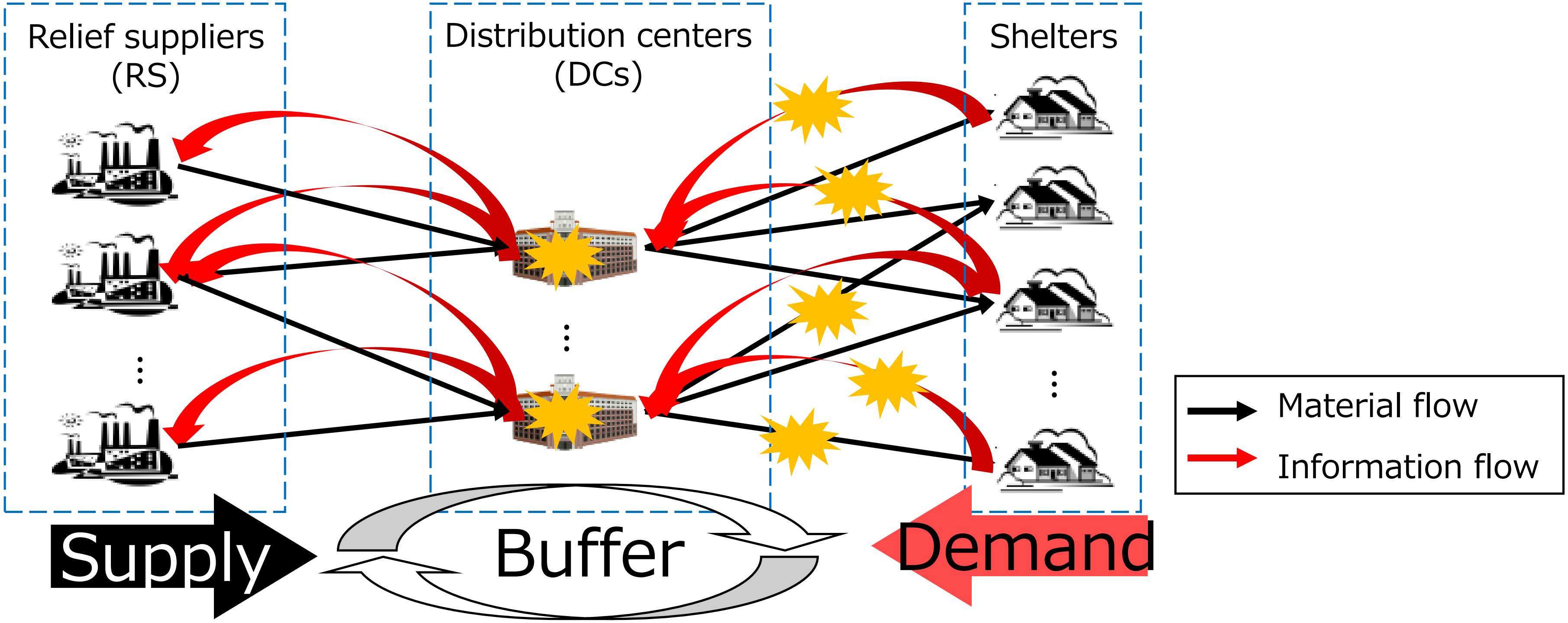
Background: Humanitarian Logistics

Humanitarian Logistics is important for minimizing the damage between the rescue and restoration period after a disaster.

Difficulties

- Node bottleneck on supply network (e.g., DCs don't work)
- Link bottleneck on material network (e.g., Cannot access)
- Link bottleneck on information network (e.g., Cannot communicate)



Kumamoto Earthquake (2016)

Many damaged roads

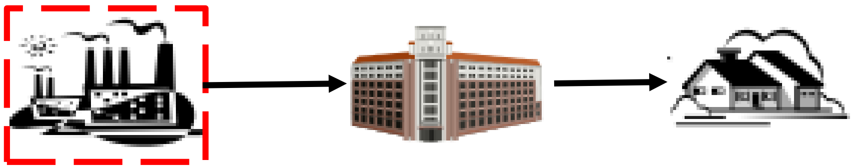
Chairs lined up to read "Paper, bread, water, SOS“

Transshipment processed inefficiently by human power

Background: Control Strategy

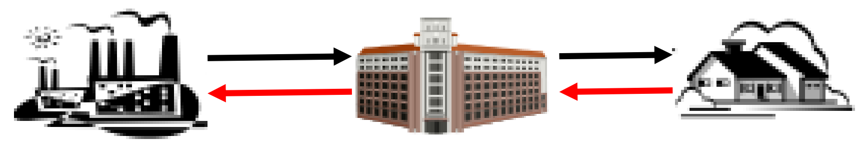
Control Strategy in Japan

[Push-mode support (sequence control)]

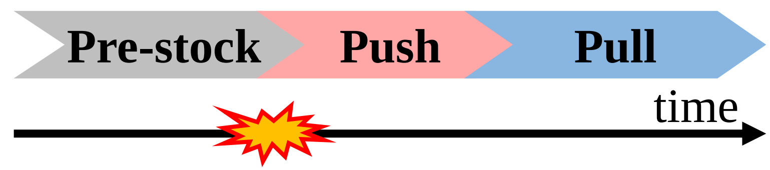


Demand forecasting

[Pull-mode support (feedback control)]



Demand feedback

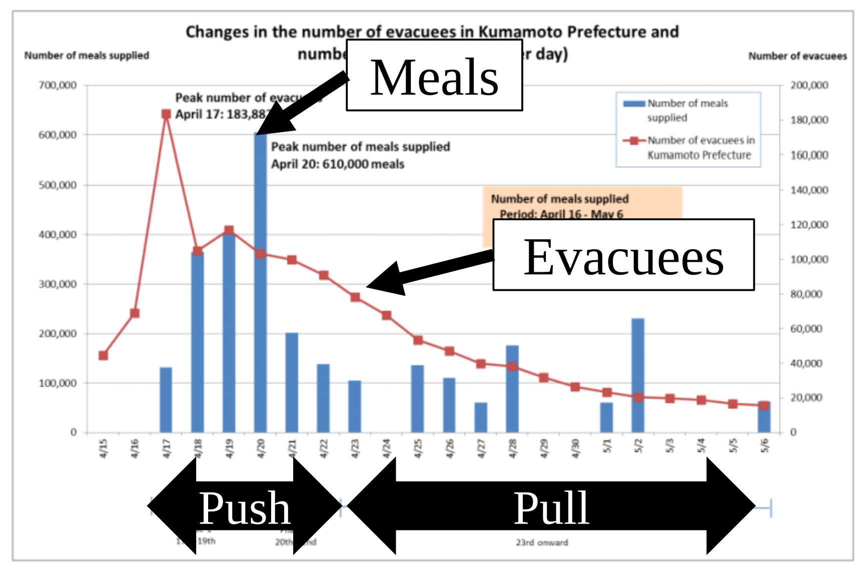


Past disasters

- Long push-mode support caused the gap between supply (meals) and demand (evacuees).

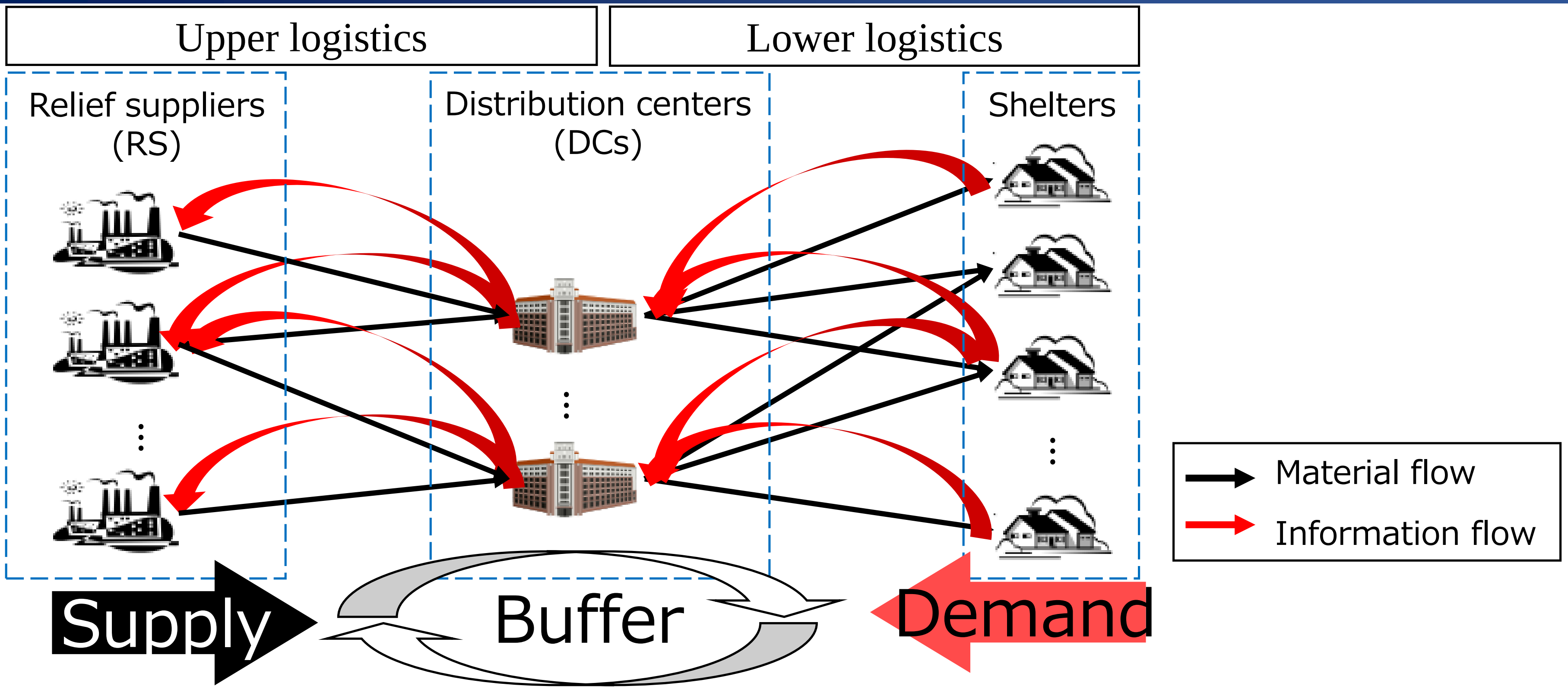
Research Question

When should the control strategy change from push to pull ?



Number of evacuees and meals supplied after the Kumamoto Earthquake

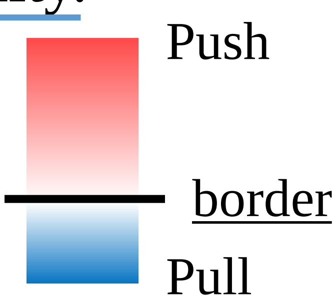
Background: Humanitarian Logistics



■ DCs can adjust gaps by holding inventories on the implicit assumption of supply and information availability.

- Push-mode
- Decentralized Pull-mode
- Centralized Pull-mode

- not available
- limited available
- fully available



Purpose and Methodology

Research Question: When should the push- be changed to pull-mode ?

- We drive the sufficient condition to change control strategy from push-mode to pull-mode.

Purpose: Mathematical properties of the optimal control strategy

- Focus on information availability
- Mathematically analyze the optimal push-mode (no information) and the optimal pull-mode (decentralized/centralized information).

Methodology

- Dynamic optimization approach using the stochastic optimal control theory considering Demand uncertainty.
- The Bayesian updating process can model two control strategies.

$$\text{(updating interval)} = \begin{cases} \infty & \text{(not available)} \\ 0\sim\infty & \text{(limited available)} \\ 0 & \text{(fully available)} \end{cases}$$

Modeling

Definition

Control variable

$S_{ij}(t)$: Supply rate from node i to node j at time t

State variable

$IN(t)$: Net inventory level in the shelter at time t

$I_i(t)$: Inventory level in node i at time t

Parameter

$D(t)$: Predicted demand rate at time t $(d\overline{D}/dt \leq 0)$

$z(t)$: Standard wiener process

T: The end of time

b: Unsatisfied cost coefficient

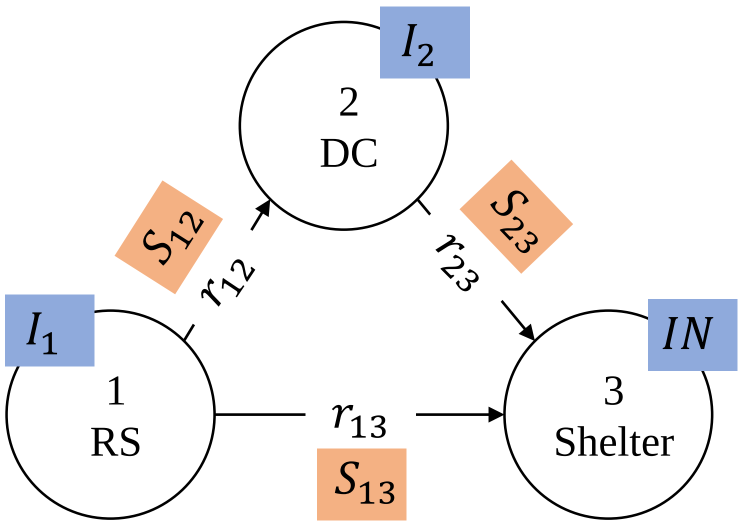
h_i': Inventory cost coefficient

$(0 < h_1' < h_2' < h_3' < b)$

c: Handling cost coefficient

r_{ij}: Lead time from node i node j

$(r_{13} < r_{12} + r_{23} = r_{123})$



Supply Chain Network (SCN)

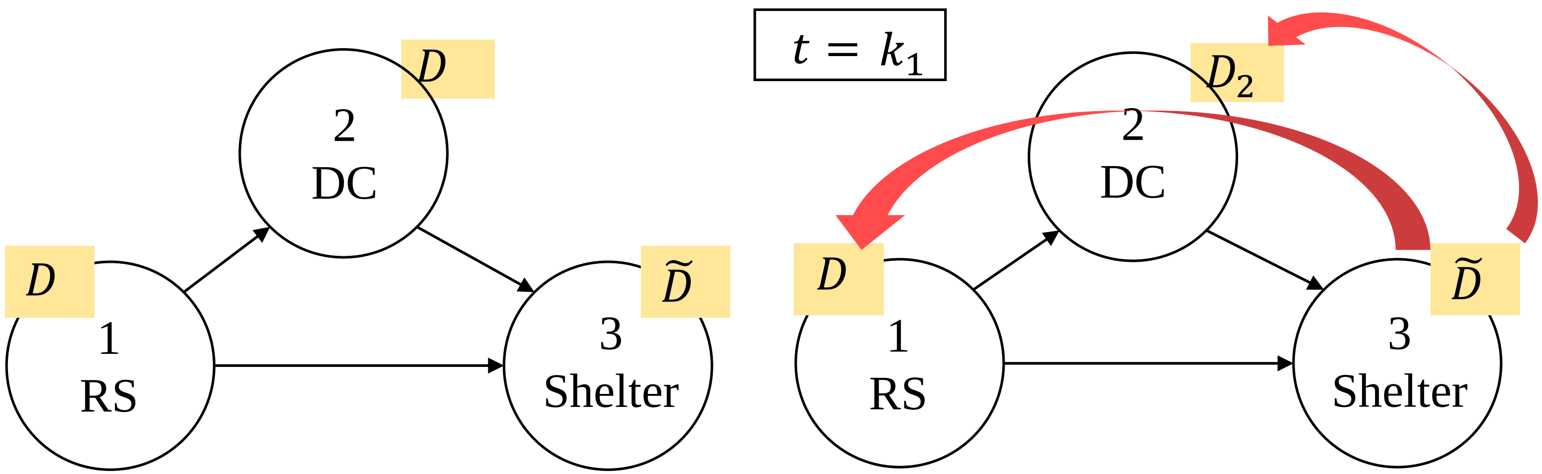
Information Updating Algorithm

- Predicted Demand $D(t)$ follows the normal distribution.

[Dynamics] $$D(t)\,dt = \overline{D}(t)dt + D^{SD}(t)dz(t)$$

[Distribution] $$D(t)dt \sim N\left(\overline{D}(t)dt, \left(D^{SD}(t)\right)^2 dt\right)$$

- $D(t)$ is updated to subjective demand $D_l(t)$ based on information $\widetilde{D}(t)$, applying the Bayesian Estimation at an interval $k_l (k_1 \geq k_2)$.
- The number of updates n increases as depot l is closer to the shelter
  $\Longrightarrow$ The Bullwhip Effect $(V[D_1(t)] \geq V[D_2(t)] \because n_1 \leq n_2)$



Push-mode support

Pull-mode support

Stochastic Optimal Control Problem

Object Function

$$\min V = E\int_0^T \left[\sum_{l\in N} TC_{IN_l}(t) + \sum_{i\in N^+} TC_{I_i}(t) + \sum_{l\in N^+} TC_{S_l}(t)\right] dt$$

[Net inventory holding cost] $TC_{IN_l}(t) = h_l\,[f_I([IN_l(t)]^+) + f_B([IN_l(t)]^-)]$

[Inventory holding cost] $TC_{I_i}(t) = h_i'[f_I(I_i(t))]$

[Inventory handling cost] $TC_{S_l}(t) = \sum_{j\in C_l} cf_S(|TS_{lj}(t)/|P_j| - S_{lj}(t - r_{ij})|)$

[Outflows at destination *j*] $$TS_{lj}(t) = \begin{cases} \sum_{i\in C_j} S_{ji}(t) & if\ j \in N^+ \\ D_l(t) & otherwise \end{cases}$$

[Cost functions] $\frac{df_X(x)}{dx} > 0,\ \frac{d^2 f_X(x)}{dx^2} > 0$, $f_X(x)$ and $df_X(x)/dx$ is continuous $\forall X \in [I, B, S]$

State Equation (Inventory Dynamics)

$$dIN_l(t) = \left[\sum_{i\in P_3} S_{i3}(t - r_{i3}) - D_l(t)\right] dt, \forall\, l \in N$$

$$D_l(t)\,dt = \overline{D}_l(t)dt + D_l^{SD}(t)dz_i(t) \quad \forall\, l \in N$$

$$\dot{I}_1(t) = 0, \dot{I}_2(t) = S_{12}(t - r_{12}) - S_{23}(t)$$

Initial Condition

$$IN_l(0) < 0, \forall\, l \in N$$

$$I_1(0) = 0, I_2(0) > 0$$

$$S_{ij}(t) = 0 \quad \forall\, t \in [-r_{ij}, 0), j \in C_i, i \in N^+$$

Stochastic Optimal Control Problem

Object Function

$$\min V = E\int_0^T \left[\sum_{l\in N} TC_{IN_l}(t) + \sum_{i\in N^+} TC_{I_i}(t) + \sum_{l\in N^+} TC_{S_l}(t)\right] dt$$

[Net inventory holding cost] $TC_{IN_l}(t) = h_l\,[f_I([IN_l(t)]^+) + f_B([IN_l(t)]^-)]$

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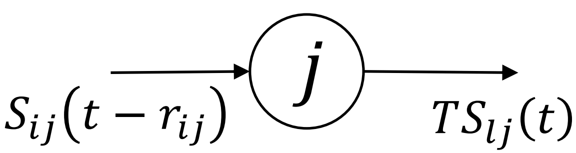
[Inventory handling cost] $TC_{S_l}(t) = \sum_{j\in C_l} cf_S(|TS_{lj}(t)/|P_j| - S_{ij}(t-r_{ij})|)$

[Outflows at destination *j*]

$$TS_{lj}(t) = \begin{cases} \sum_{i\in C_j} S_{ji}(t) & if\ j \in N^+ \\ D_l(t) & otherwise \end{cases}$$

- $TC_{S_l}(t)$: Changes in the inventory level in node l
  - $\min TC_{S_l}(t) = 0$ (∴ no transshipment)
  - $TC_{S_l}(t) \neq 0$ means that the inventory level should change (∴ handling cost).
  - $\min TC_{S_l}(t)$ means supply constraints for node l



$S_{ij}(t) = 0 \quad \forall\, t \in [-r_{ij}, 0), j \in C_i, i \in N^+$

$\dot{I}_1(t) = 0, \dot{I}_2(t) = S_{12}(t-r_{12}) - S_{23}(t)$

Mathematical Analysis

- Optimal Push-mode Support -

Assumption

<u>Cost functions</u>

$$f_I(x) = f_B(x) = f_S(x) = x^{\alpha}, \alpha > 1$$

<u>Assumption</u>

1. Let $t = T$ be the time when demand becomes 0, $\overline{D}_l(T) = 0$.
2. Demand decreases constantly over time, $d\overline{D}_l(t)/dt = \dot{\overline{D}}_l < 0$.
3. The inventory holding cost at the shelter is twice that at the RS, $h'_{IN} = 1/2h'_1$.
4. The DC is not ready after a disaster, $S_{23}(t) = 0 \ \forall t \in [0, r_{12})$.

The following mathematical properties will be proved:

Lemma 1. There is no need to pre-store at DC and to add stock after a disaster, $I^*(t) = 0$.

Lemma 2. "Direct Supply" is effective, $E[S^*_{12}(t)] < E[S^*_{13}(t)]$.

Theorem 1. DC is unnecessary for sequence control strategy.

Theorem 2. Maintaining inventory at the shelter is the optimal when the penalty cost is sufficiently high, $IN^*(t) > 0 \ if \ b \to \infty$.

Optimal Control : $I(t)$

Optimal Inventory

$$I^*(t) = I(0)\exp\left(-t\sqrt{\frac{h_2'}{c}}\right)\frac{1+y_I^2(t)}{1+y_I^2(r_{12})} \qquad t\in(r_{12},T]$$

$$y_I(t) = \exp\left((t-T)\sqrt{\frac{h_2'}{c}}\right)$$

$$V^* = V(I^*, IN^*, S^*)$$

- Positive inventory level $I^*(t) > \mathbf{0}$
- Decreasing function $\dot{I}^*(t) = -I(0)\exp\left(-t\sqrt{\frac{h_2'}{c}}\right)\frac{1-y_I(t)}{1+y_I(r_{12})} < 0$
- The limit value is 0 $\lim_{T\to\infty} I^*(T) = I(0)\cdot 0\cdot\frac{2}{1+0} = 0$

- $I(t)$ asymptotically approaches 0 from initial value $I(0) > 0.$

  Pre-stock

  $\rightarrow$ Solve $I^*(0) = \text{argmin}_{I(0)}\, V^*$

$$\frac{\partial V^*}{\partial I(0)} = 2h_2' I(0)\left[r_{12} + \int_{r_{12}}^{T}\exp\left(-2s\sqrt{\frac{h_2'}{c}}\right)\frac{1+y_{IN}^4(s)}{(1+y_{IN}^2(r_{12}))^2}ds\right]$$

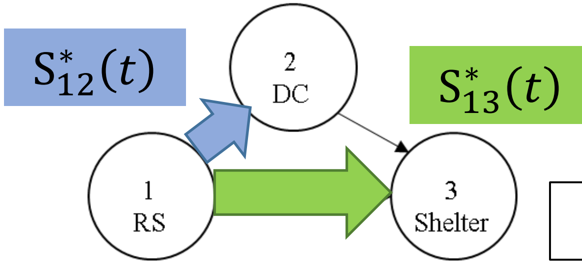
> 0, therefore $I^*(0) = 0$

Lemma 1

There is no need to pre-store at DC and to add stock, $I^*(t) = 0$.

Optimal Control : $S_{1j}(t)$

<u>Optimal supply rate from RS</u>



$r_{13} < r_{123}$

$$S_{12}^{*}(t) = \hat{S}(t + r_{123})$$
$$S_{13}^{*}(t) = \hat{S}(t + r_{13})$$

$t \in [r_{123} - r_{13}, T - r_{123})$

$$\hat{S}(t) = -\exp\left[(r_{123} - t)\sqrt{\frac{2h'_{IN}}{c}}\right]\sqrt{\frac{h'_{IN}}{2c}}\frac{1-y_{IN}^2(t)}{1+y_{IN}^2(r_{123})}\left[IN(r_{123}) - \int_{r_{123}}^{t}\frac{D^{SD}(t)}{U(t)}dz(t) + \frac{c\dot{\overline{D}}}{2h'_{IN}}\right] + \frac{\overline{D}(t)}{2}$$

Differentiate $E\left[\hat{S}(t)\right]$ as follows:

$$\frac{dE\left[\hat{S}(t)\right]}{dt} = \exp\left[(r_{123} - t)\sqrt{\frac{2h'_{IN}}{c}}\right]\frac{h'_{IN}}{c}\frac{1+y_{IN}^2(t)}{1+y_{IN}^2(r_{123})}E\left[IN(r_{123})\right] + \frac{\dot{\overline{D}}}{2} \quad < 0$$

We obtain,

$$E\left[IN(r_{123})\right] = U(r_{123})E\left[IN(r_{13})\right] - c\overline{D}(r_{123})\frac{1-y_{IN}^2(r_{13})}{\psi^{+} - \psi^{-}y_{IN}^2(r_{13})} - \frac{c\dot{\overline{D}}}{2h'_{IN}}\left[U(r_{123}) - 1 + \psi_{IN}(r_{123})\frac{1-y_{IN}^2(r_{13})}{\psi^{+} - \psi^{-}y_{IN}^2(r_{13})}\right]$$

$$U(r_{123}) - 1 + \psi_{IN}(r_{123})\frac{1-y_{IN}^2(r_{13})}{\psi^{+} - \psi^{-}y_{IN}^2(r_{13})} = -\frac{2\sqrt{ch'_{IN}}}{\psi^{+} - \psi^{-}y_{IN}^2(r_{13})}\left(y_{IN}(r_{13}) - 1\right)^2 < 0$$

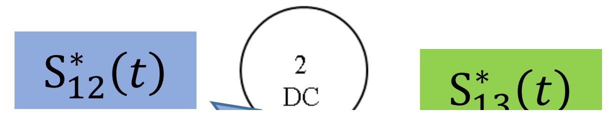
When $dE\left[\hat{S}(t)\right]/dt < 0$, we obtain $E[S_{12}^{*}(t)] < E[S_{13}^{*}(t)]$ ($\because r_{13} < r_{123}$)

<u>Lemma 2</u>

"Direct Supply" is effective, $E[S_{12}^{*}(t)] < E[S_{13}^{*}(t)]$.

Optimal Control : $S_{1j}(t)$

Optimal supply rate from RS



Lemma 1

There is no need to pre-store at DC and to add stock, $I^{*}(t) = 0$.

Lemma 2

"Direct Supply" is effective, $E[S_{12}^{*}(t)] < E[S_{13}^{*}(t)]$.

Theorem 1

DC is unnecessary for sequence control strategy.

Optimal Control : $IN(t)$

<u>Optimal net inventory</u>

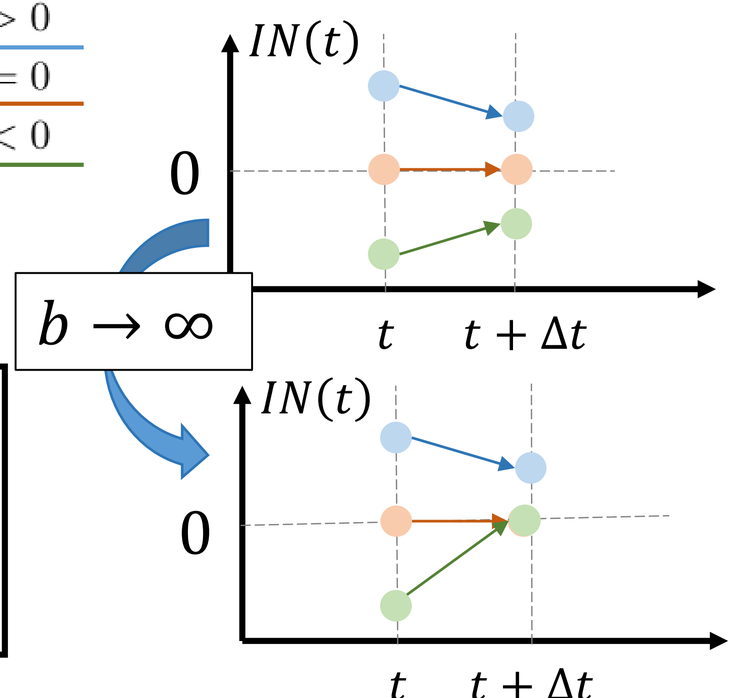
[Dynamics] $$dIN(t) = -\sqrt{\frac{2h'_{IN}}{c}\frac{1-y_{IN}^2(t)}{1+y_{IN}^2(t)}}\left[IN(t)-\mu^{\infty}\right]dt - D^{SD}(t)\,dz(t)$$

[Optimal] $$IN^*(t) = U(t)\left[IN(r_{123}) - \int_{r_{123}}^{t}\frac{D^{SD}(r)}{U(r)}dz(r)\right] \qquad t\in(r_{123},T]$$

- Long-term expected value is 0 $\quad \mu^{\infty} = \lim_{T\to\infty} E[IN(T)] = 0\cdot E[IN(r_{123})] = 0$
- Regression speed < 0 $\quad -\sqrt{\frac{2h'_{IN}}{c}\frac{1-y_{IN}^2(t)}{1+y_{IN}^2(t)}} < 0$

- Dynamics of $IN(t)$ is $\quad dIN(t)\begin{cases} <0 & IN(t)>0 \\ =0 & IN(t)=0 \\ >0 & IN(t)<0 \end{cases}$
- Assume $b\to\infty$

$$h'_{IN} = \begin{cases} h'_3 & if\ IN(t)\ge 0 \\ b & otherwise \end{cases}$$



Theorem 2

Maintaining inventory at the shelter is the optimal when the penalty cost is sufficiently high, $IN^*(t) > 0\ if\ b\to\infty$.

Numerical Analysis

- Optimal Pull-mode Support -

Settings

Analysis method

- Comparing the objective functions of the push V^{push} and pull $V^{pull}(k_1, k_2)$ with the Monte Carlo simulation.

Parameter setting

- Prediction demand $\overline{D}(t) = -0.15(t - T)$.
- The number of simulations is 5,000.
- case-a「$R[z_1(t), z_2(t)] = 1$」, case-b「$R[z_1(t), z_2(t)] \neq 1$」

RS and DC share prediction errors / not share

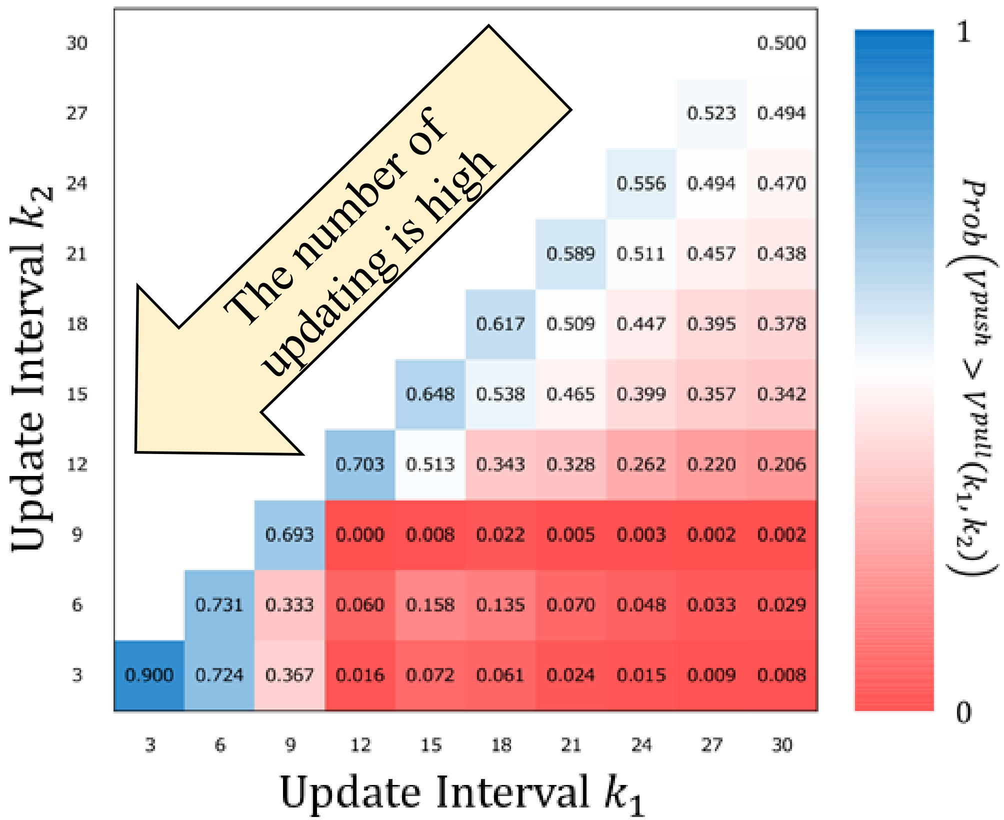
Parameter setting

t	$[0, 10)$	$[10, 20)$	$[20, 30]$
$\tilde{D}(t)$	4	2	1
$\overline{D}(t)$	$-0.15(t - T)$		
$\{D^{SD}(t), \sigma(t)\}$	$\{100, 50\}$		

$\{r_{12}, r_{23}, r_{13}\}$	$\{h'_2, h'_3, b\}$	c	$IN(0)$
$\{3, 2, 4\}$	$\{0.5, 0.7, 1\}$	10	-200

Results : Comparing Push and Pull

$R[z_1(t), z_2(t)] = 1$(case–a)



- The sufficient conditions for control strategy change (push-mode to pull-mode) is $k_1 = k_2$, that is **the centralized information system is restored.**

- Under the decentralized information system, pull-mode may not be effective.

Conclusion

Conclusion

<u>Summary</u>

- Dynamic optimization approach analyzed the mathematical properties of the optimal control strategy.
  - Push-mode: direct supply from the RS to the shelter is optimal transportation strategy. DC should not be used.
  - Pull-mode: the sufficient condition for pull-mode to be optimal is to restore the centralized information system. Otherwise, pull-mode may make a not always good result.

<u>Future work</u>

- General network analysis (e.g., many-to-many network) can consider significant elements such as *the single point of failure*.
- Considering optimal day-to-day recovery dynamics.

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Thank you for your attention.

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