# Dynamics in Traffic Analysis

September, 22 2019

Summer School by Prof. Hato

Graduate School of Information Sciences, Tohoku University
Masao Kuwahara

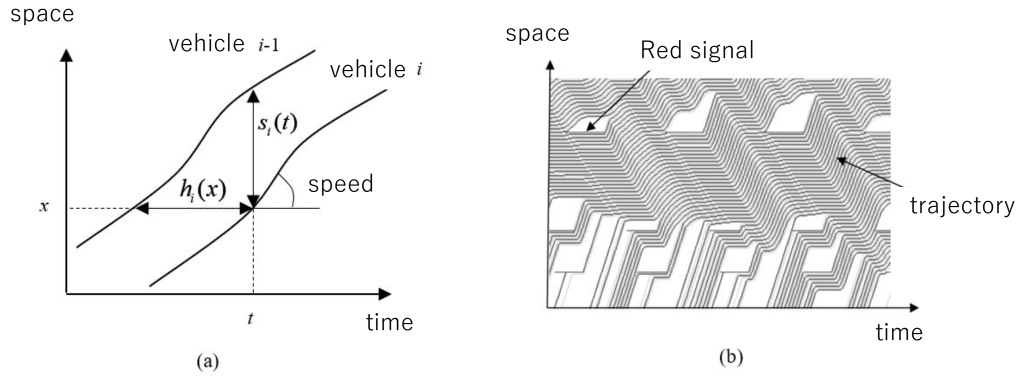
1

## Outline

| | | |
|---|---|---|
| Flow conservation<br>Fundamental diagram<br><br>3-dim flow representation | Time-space diagram<br><br>Cumulative figure | Fundamentals |
| Kinematic wave theory<br>differential equation<br>Lower envelop principle<br>Variational theory | Queueing Theory<br>Point / Physical Queue | Theory |
| Simulation<br>CTM (Cell Transmission Model)<br>Block-Density method<br><br>Traffic State Estimation | Dynamic network analysis<br>DUO, DUE<br><br>Departure time choice<br>Dynamic marginal cost | Applied Theory |

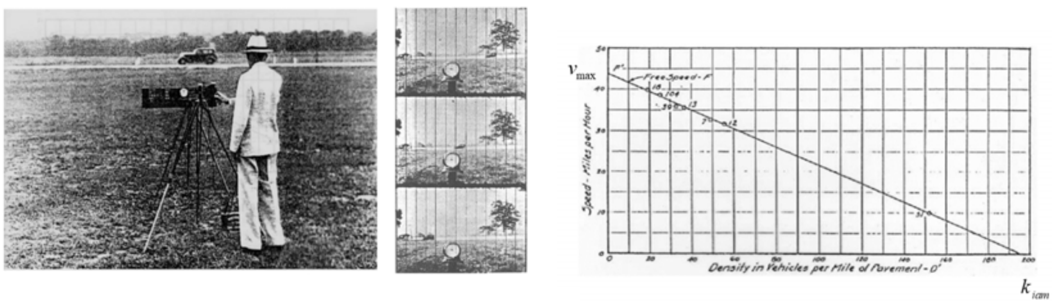
2

Time-Space Diagram

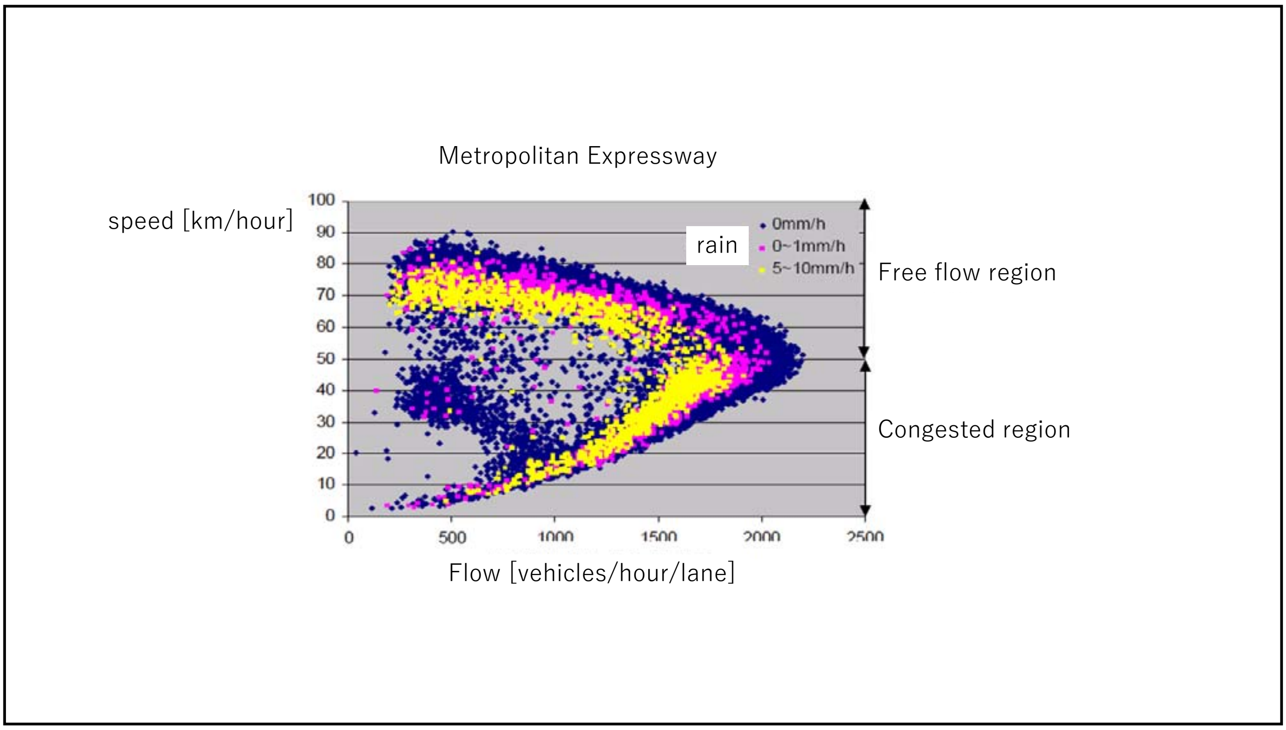





Fundamental Diagram - Flow, Density, Speed

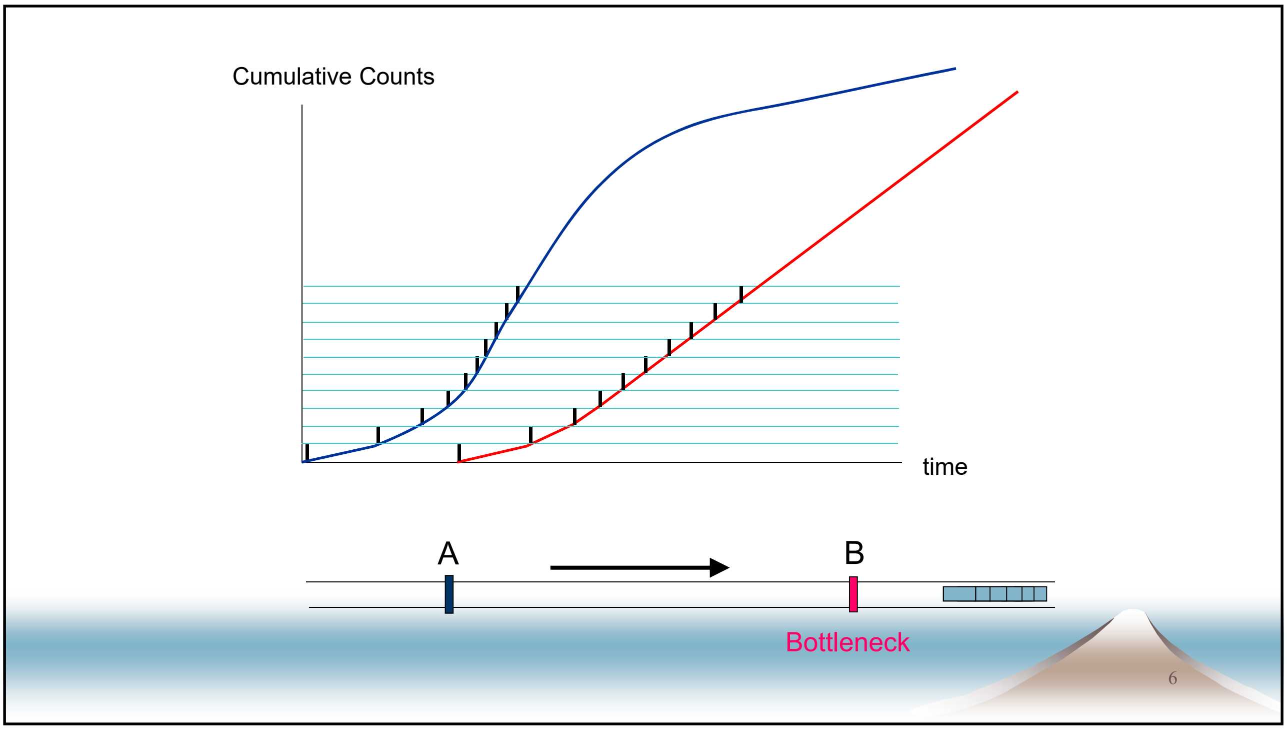



Greenshield’s Measurement by 16 mm Camera in Michigan in 1933
(Kuhne 2011)



Metropolitan Expressway
speed [km/hour]
100
90
80
70
60
50
40
30
20
10
0
0
500
1000
1500
2000
2500
Flow [vehicles/hour/lane]
rain
0mm/h
0~1mm/h
5~10mm/h
Free flow region
Congested region


5

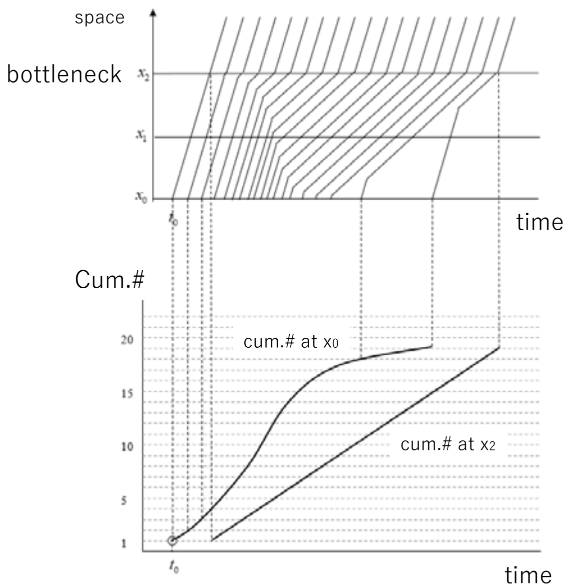


Cumulative Counts
time
A
B
Bottleneck
6


6

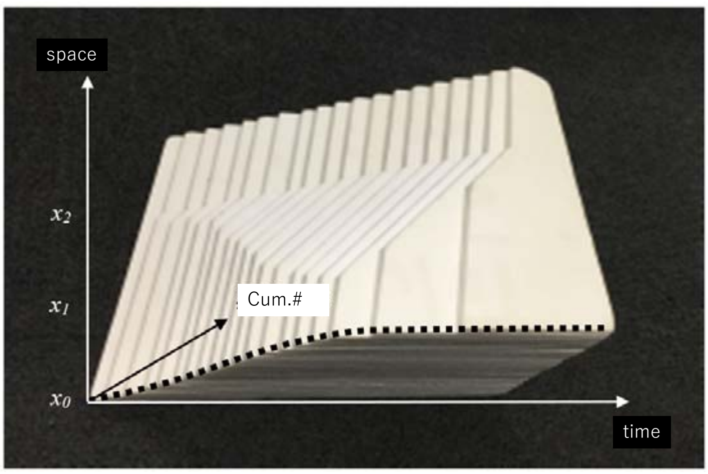
Time-space diagram

Cumulative figure



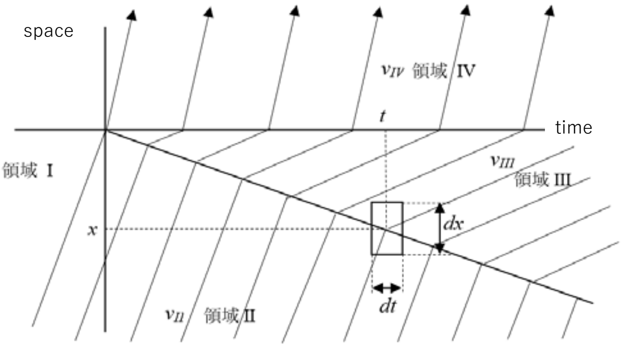



Three-dimensional Flow Representation



### Kinematic Wave Theory

Flow, density, speed, cumulative count on time-space (x,t)
q(x,t) k(x,t) v(x,t) N(x,t)




$$q(x,t)=\frac{\partial N(x,t)}{\partial t}$$

$$k(x,t)=-\frac{\partial N(x,t)}{\partial x}$$



### Kinematic Wave Theory

Flow Conservation

$$\frac{\partial q(x,t)}{\partial x}=\frac{\partial^2 N(x,t)}{\partial t \partial x}=-\frac{\partial k(x,t)}{\partial t} \quad \rightarrow \quad \frac{\partial q(x,t)}{\partial x}+\frac{\partial k(x,t)}{\partial t}=0$$

Fundamental diagram (assume to exist any (x,t))

$$k=k^*(q,x,t)$$
$$q=q^*(k,x,t)$$

$k^*$, $q^*$ = functions

## Kinematic Wave Theory

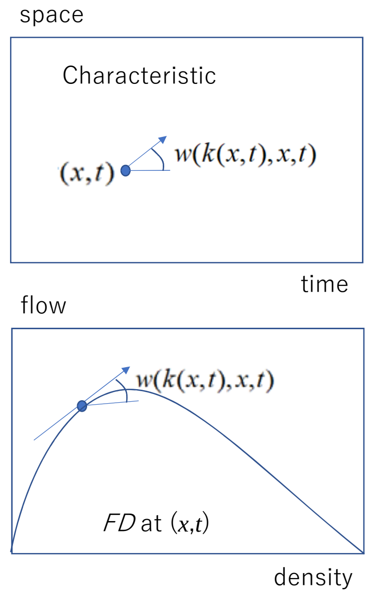
Characteristic

$$
\begin{aligned}
dk(x,t) &= \frac{\partial k(x,t)}{\partial t}dt + \frac{\partial k(x,t)}{\partial x}dx \\
&= \left(\frac{\partial k(x,t)}{\partial t}\frac{dt}{dx} + \frac{\partial k(x,t)}{\partial x}\right)dx \\
&= \left(-\frac{\partial q(x,t)}{\partial x}\frac{\partial k(x,t)}{\partial q(x,t)} + \frac{\partial k(x,t)}{\partial x}\right)dx = 0
\end{aligned}
$$

Flow conservation

Wave speed

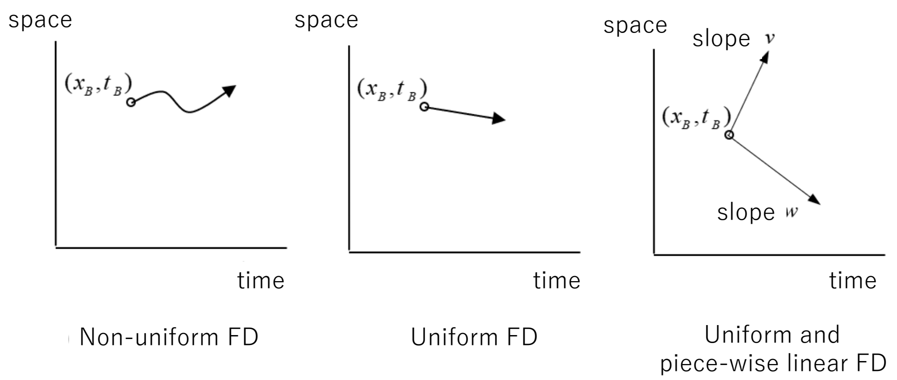
$$\frac{dx}{dt} = \frac{\partial q^*(k(x,t),x,t)}{\partial k(x,t)} = w(k(x,t),x,t)$$




11

## Kinematic Wave Theory

Characteristic Curve (trajectory on which flow, density and speed are constant)



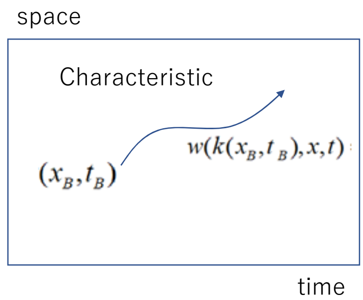

12

## Kinematic Wave Theory

Solving the differential equation along Characteristic

$$\frac{dx}{dt} = w(k(x,t),x,t) \quad \rightarrow \quad dx = w(k(x,t),x,t)dt$$

$$x(t) = x_B + \int_{t_B}^{t} w(k(x_B,t_B),x,t)dt$$

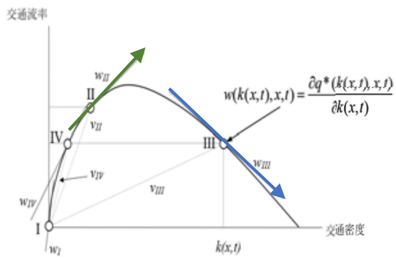



- The initial density $k(x_B,t_B)$ is given and does not change along the characteristic curve.
- Thus, wave speed can be known at any $(x,t)$, if the fundamental diagram is given everywhere.



## Kinematic Wave Theory

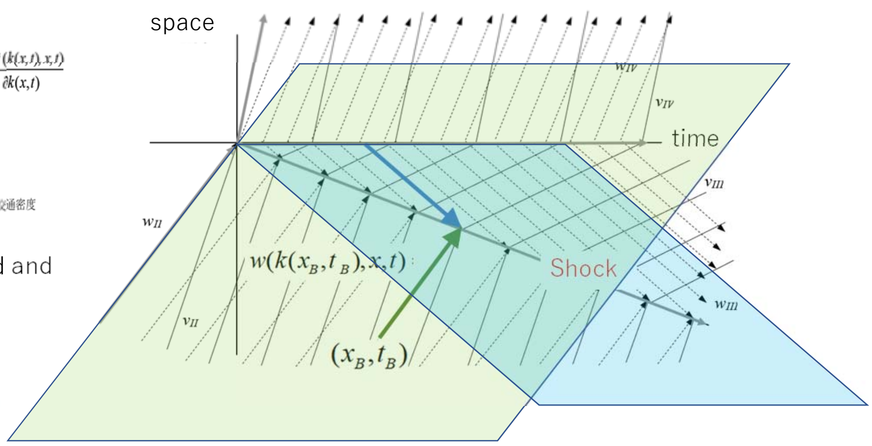
Lower Envelop Principle by Newell (1993), Luke (1972)




On the shock, the wave speed and the density suddenly change.

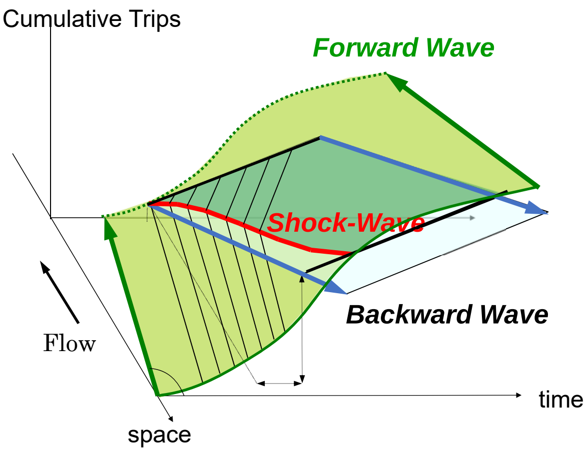
Difficult the integration:

$$x(t) = x_B + \int_{t_B}^{t} w(k(x_B,t_B),x,t)dt$$



Kinematic Wave Theory

Lower Envelop Principle by Newell (1993), Luke (1972)

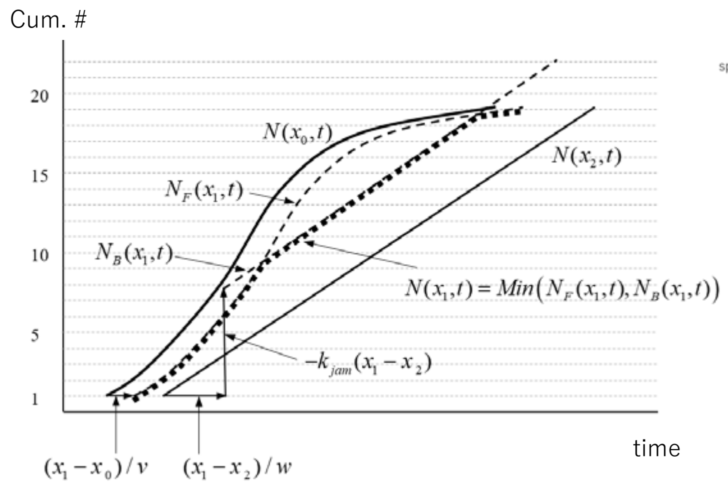


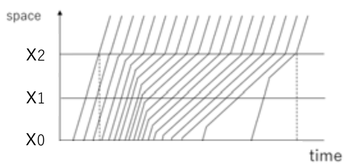


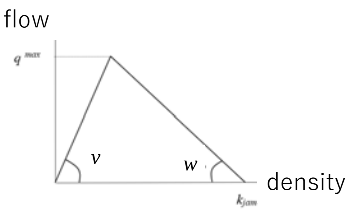


Kinematic Wave Theory

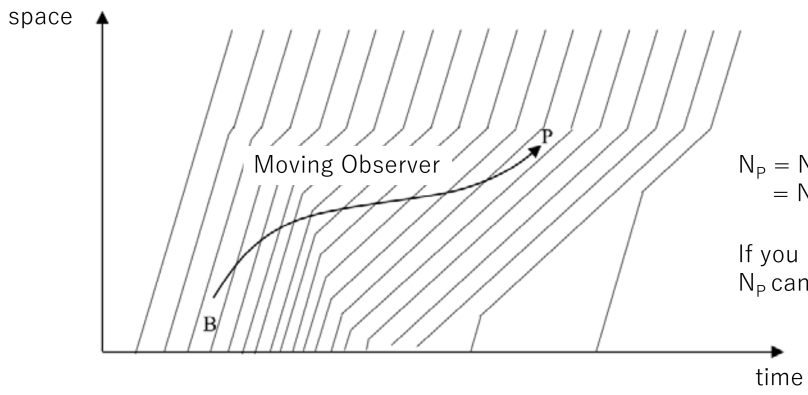
Lower Envelop Principle by Newell (1993), Luke (1972)



Variational Theory : Moving Observer

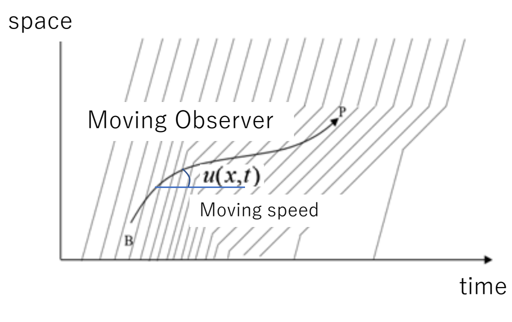



$N_P = N_B + \Delta BP$ (# of overtaken)
$= N_B + 10$

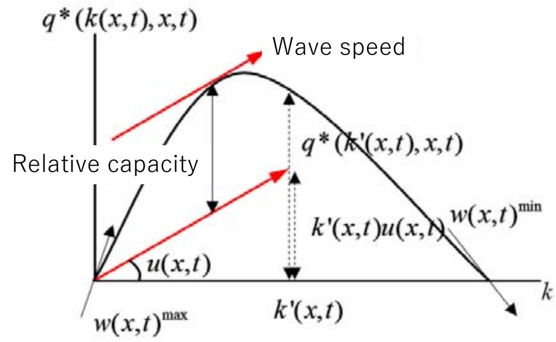
If you know $\Delta BP$ ,
$N_P$ can be evaluated.



Variational Theory : Relative Capacity ( = max # of overtaken)



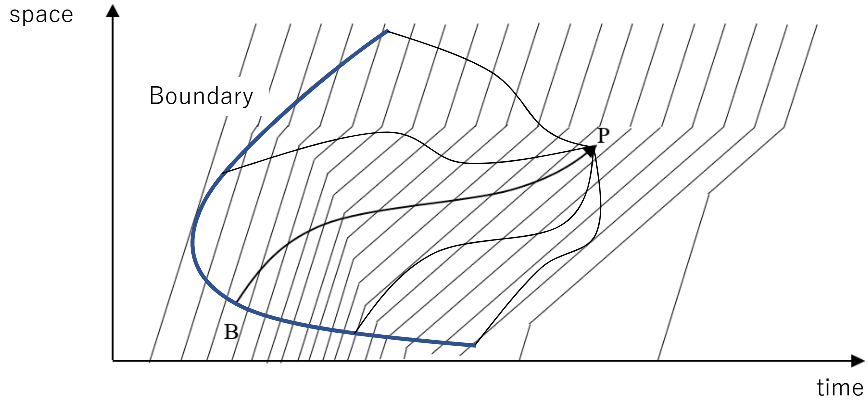

$N_P \leq N_B + \Delta_{max}BP$ (relative capacity)




Relative Capacity $= \sup_{k(x,t)}\left(q^*(k(x,t),x,t) - k(x,t)u(x,t)\right)$

Variational Theory : Evaluation of $N_P$

$$N_P = \mathrm{Min}_B \{N_B + \Delta_{max} BP\}$$

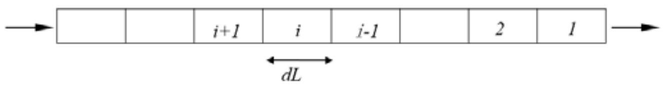


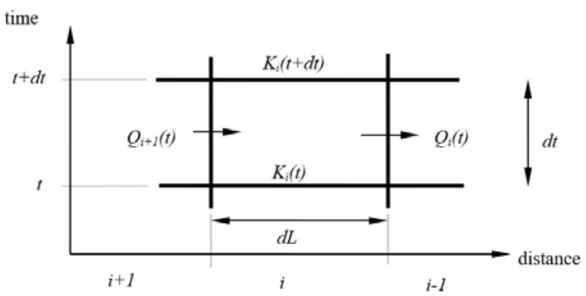

At boundary points, the cumulative counts are known.



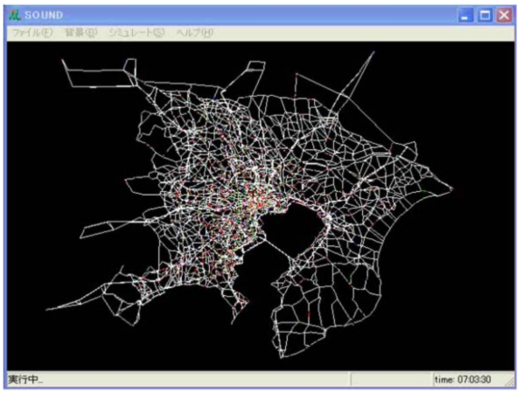
CTM (Cell Transmission) and Block Density Models

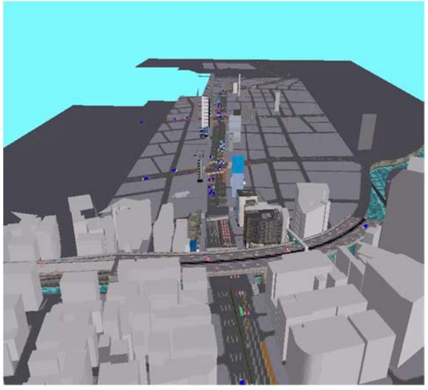
based on Flow Conservation & Fundamental Diagram
(Kinematic Wave and Lower Envelop)



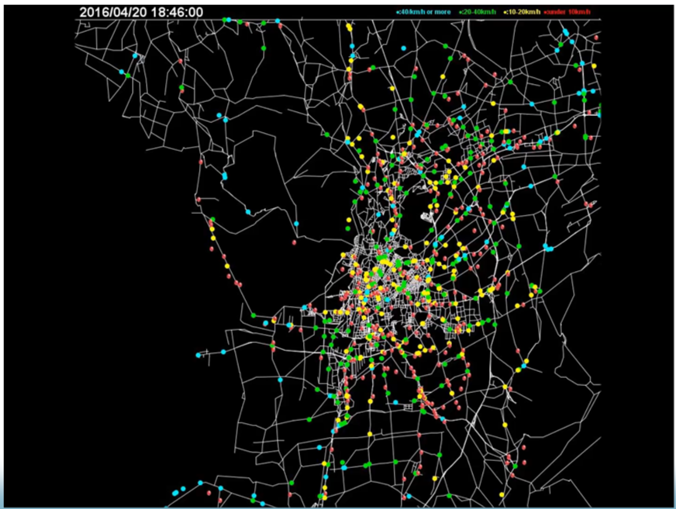
Simulation : CTM (Cell Transmission) and Block Density Models



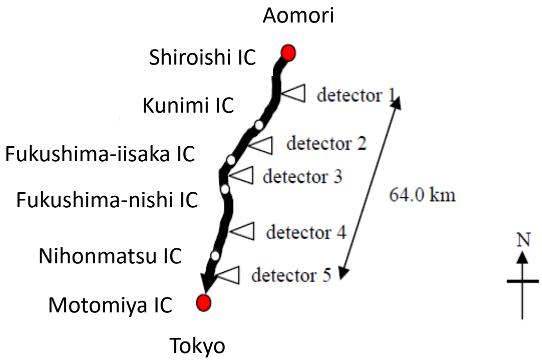





Traffic State Estimation : Application to Intercity Motorway



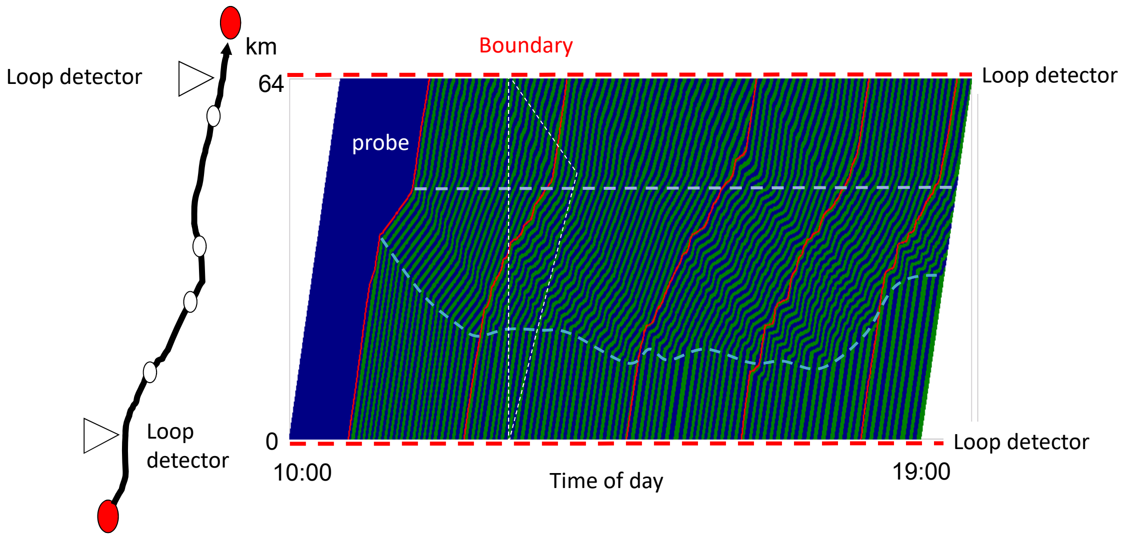
## Traffic State Estimation : Application to Intercity Motorway



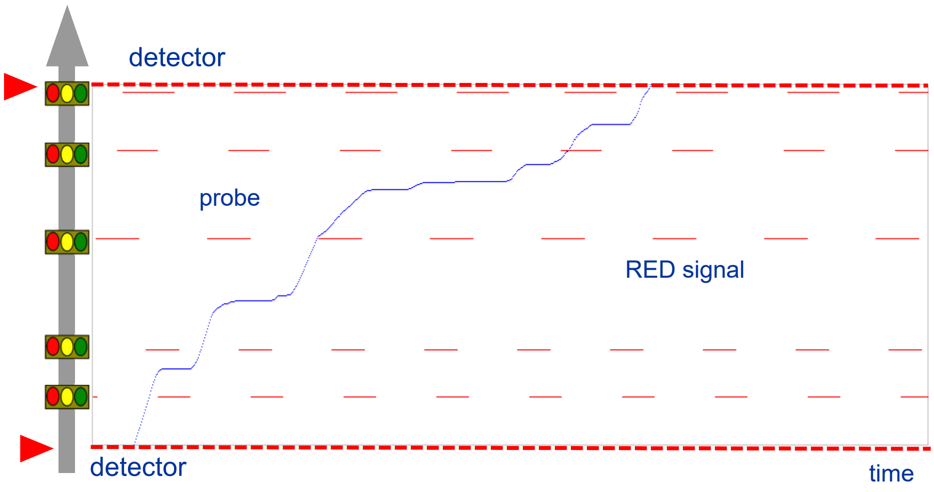





## Traffic State Estimation : Application to Intercity Motorway

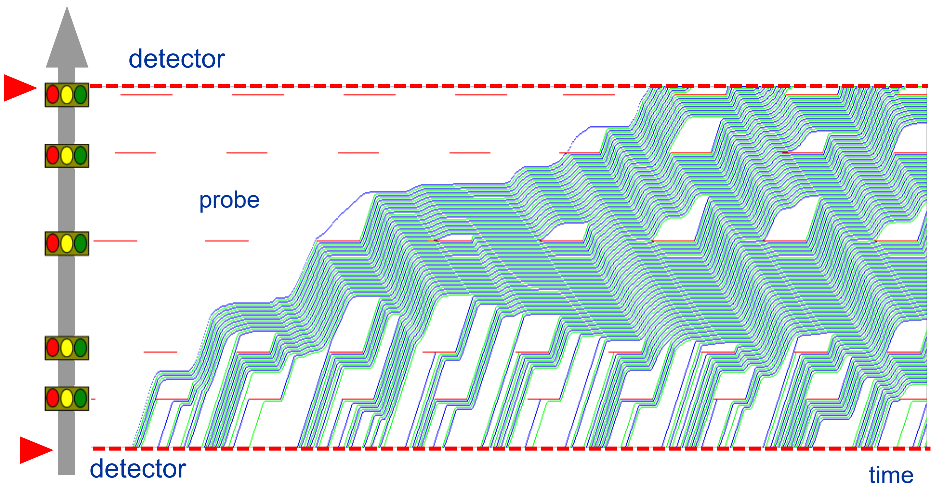


Traffic State Estimation: Application to Surface Streets

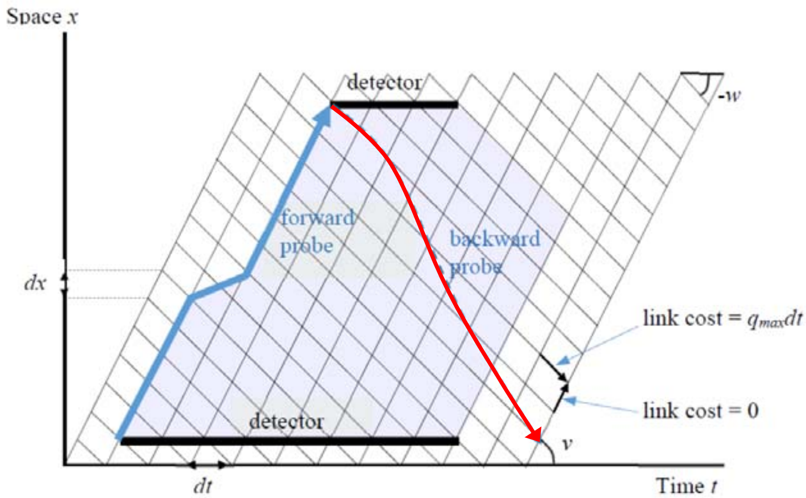





Traffic State Estimation: Application to Surface Streets

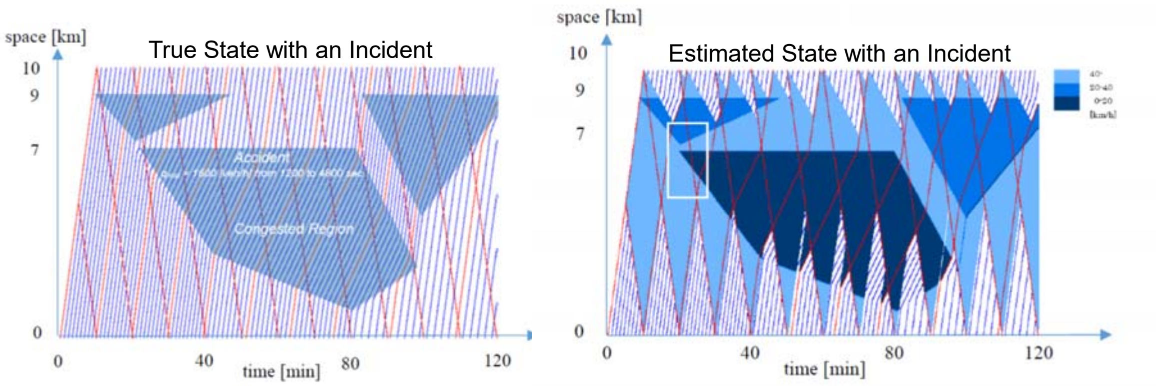


Traffic State Estimation: using Backward probe






Traffic State Estimation: using Backward probe




Backward probes can estimate traffic states

- even under an incidence by observing the capacity at the incident site
- even with in / out flows in a middle of the study section

Traffic State Estimation : Two-dimensional Extension

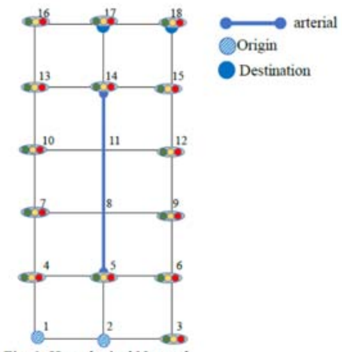
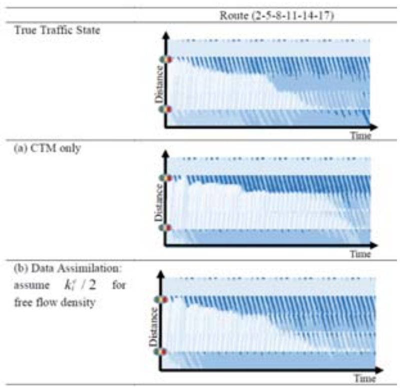



Fig. 3 Hypothetical Network

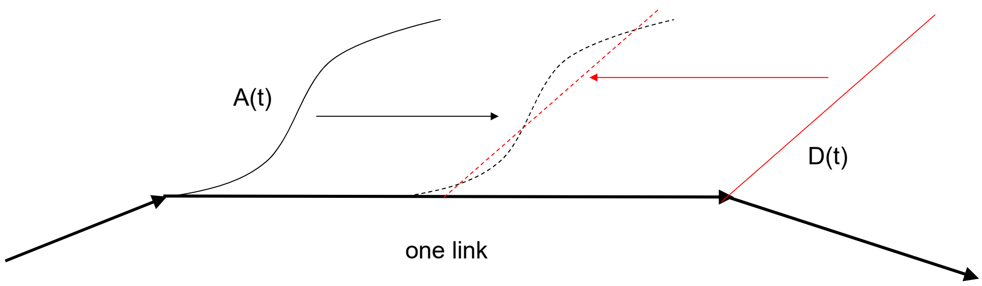



Two-dimensional extension was proposed using CTM.
It can estimate the OD demand and FD parameters simultaneously with the traffic state.



Dynamic Network Analysis

Physical Queue is analysed using Lower Envelop Principle



Dynamic Network Analysis

Route Choice Principle

DUO ( Dynamic User Optimal) = Reactive    Decomposition by absolute time
= choose the best (shortest) route based on current traffic condition

DUE ( Dynamic User Equilibrium) = Predictive
= choose the best (shortest) route actually experienced

Queueing

Point Queue

Physical Queue (congestion propagation) ← Kinematic Wave
Lower Envelop Principle

OD demand

One-to-Many OD + DUE    Decomposition by departure time from a single origin

Many-to-Many OD + DUE    Tough Problem!



Dynamic Network Analysis : historical development

| | **DUO (reactive)** | | **DUE (predictive)** | |
|---|---|---|---|---|
| | Point Queue | Physical Queue | Point Queue | Physical Queue |
| One-to-Many OD | done Simulation (CTM etc) | done Simulation (CTM etc) | done 1993 Decomposition by departure time | done 2005 Decomposition by departure time Lower Envelop |
| Many-to-Many OD | done 1997 Decomposition by absolute time | done 2001 Decomposition by absolute time Lower Envelop | Tough! | Tough! |

Future Issues on Dynamics in Traffic Analysis

Flow modeling by lanes, by vehicle types
- different flow conditions by lanes
- multi modes including motorcycles, bikes, pedestrians
- heterogeneity of drivers
- anticipation, viscosity for smoother transitions

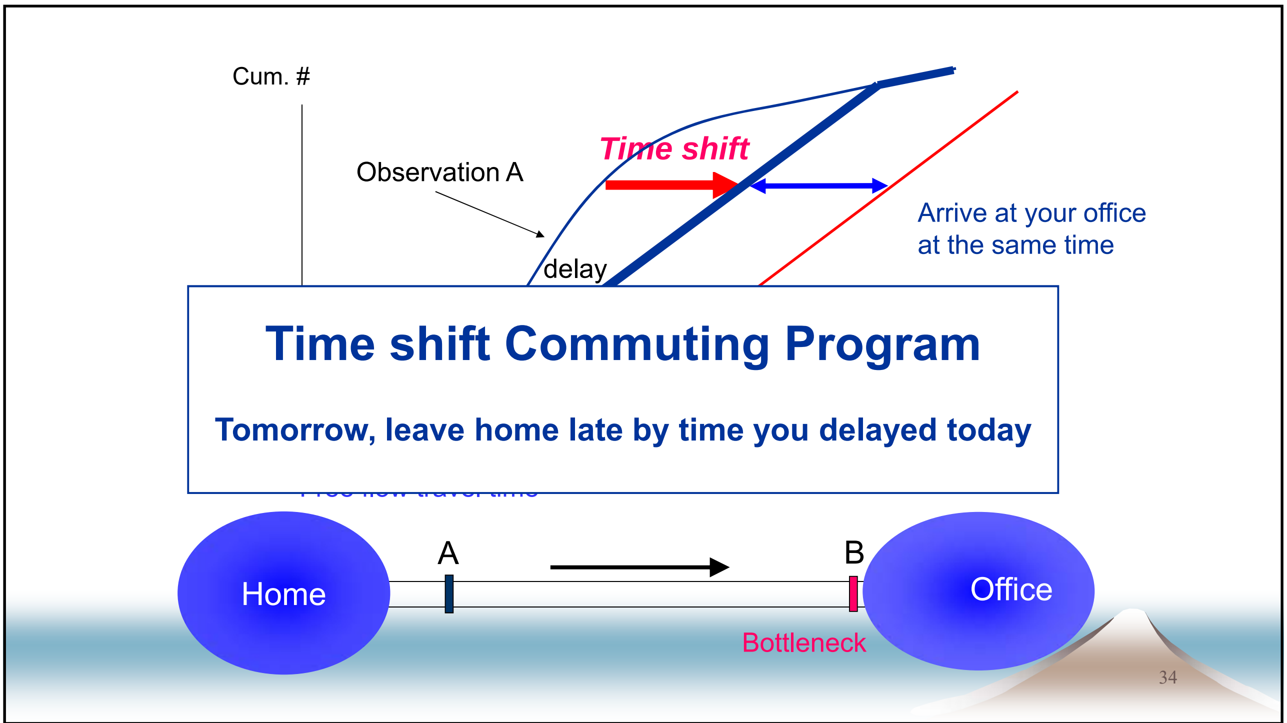
Traffic state estimation by fusing various kinds of data
- more data will be available under ADAS, ADS environment
- more needs in disaster/ incident conditions (natural disasters, big events)

DUE with Many-to-Many OD
- one of the toughest problem kept for young researchers!

Applications to control traffic, mitigate disaster, information provision, etc.

ADAS : Advanced Driver Assistance Systems(ADAS)
ADS : Automated Driving System

33




34