2022/09/25
#2 BinN Research Seminar

Characteristic of transport network based on the eigenvalue analysis

固有値解析に基づく交通ネットワークの特性分析

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Eigenvalue-based evaluation for transportation network

Eigenvalue analysis for transportation network 固有値解析を用いた交通ネットワークの分析

Fiedler vector : Algebraic connectivity defined by the second smallest eigenvalue of the Laplacian matrix (degree matrix - adjacency matrix), and the corresponding eigenvector (Fiedler vector) to evaluate road network connectivity

Laplacian 行列の第二最小固有値によって定義される代数的連結度，それに対応する固有ベクトルを用いて道路ネットワークの連結性，接続性を評価

Ex. 中南ら(2018), Bell et al. (2017)

Wang and Van Mieghem (2008) studied on improving network connectivity by adding links using algebraic connectivity index.

Directed network

Eigenvector centrality : Port accessibility is evaluated by eigenvector centrality indices for delivery cargo network. (Wang and Cullinane, 2008)

物流ネットワークのアクセシビリティ評価

Leading eigenvalue : Identification of single additional links that contribute significantly to improve port network connectivity (Cheung et al, 2020)

港湾ネットワークの連結性向上のための追加航路の特定

Topological analysis

Typical examples of topological based road network evaluation

- *Network efficiency*
  - Latora and Marchiori (2001) defined a measure of information exchange which is the average across all node pairs of the reciprocal of distance.
    全ノード間距離の逆数の平均によってネットワーク効率性を定義
  - Mattson and Jenelius (2015) presented a global efficiency index, which indicates how direct the connections are between all node pairs by comparing the Euclidean distances with the shortest network distances.
    ユークリッド直線距離に基づく最短距離とネットワーク効率性を比較することにより，グローバル効率性指標を定義
- *Node centrality*
  - A measure of how important each node is on the network.
    各ノードがネットワーク内でどれほど重要であるかの指標
  - There are several centrality indicators depending on what is defined as “important”.
    何をもって重要とするかによって，さまざまな中心性指標が定義されている

    Ex. Degree centrality (Proctor and Loomins, 1951), Closeness centrality (Beauchamp, 1965), Eigenvector centrality (Bonacich, 1972), Betweenness centrality (Freeman, 1977)

Eigenvector Centrality (Bonacich, 1972)

- If a node adjacent to the important nodes, the centrality of that node is also large.
  重要なノードと接続するノードが隣接している場合，そのノードの中心性も大きくなる
- Important node pushes up the centrality of adjacent nodes.
  重要なノードは隣接ノードの中心性を増加させる

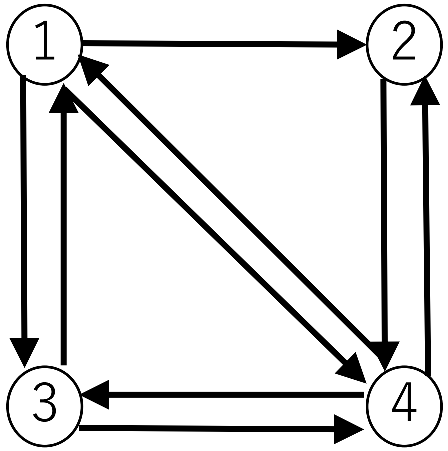
$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t)$$

$$x_i(t+1) = \sum_{j=1}^{N} A_{ij} x_j(t)$$

x : the magnitude of effect
$\mathbf{A}$: adjacency matrix
N: the number of nodes

1st iteration :

$$\begin{pmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{pmatrix}$$



This iteration converges to the largest eigenvalue λ_{max}

$$\mathbf{A}\mathbf{x} = \lambda_{max}\mathbf{x} \qquad x_i > 0\ (1 \le i \le N)$$

λ_{max}: the largest eigenvalue of $\mathbf{A}$

Eigenvector centrality is obtained by the eigenvector corresponding to the largest eigenvalue of the adjacency matrix.

Construct of weighted network

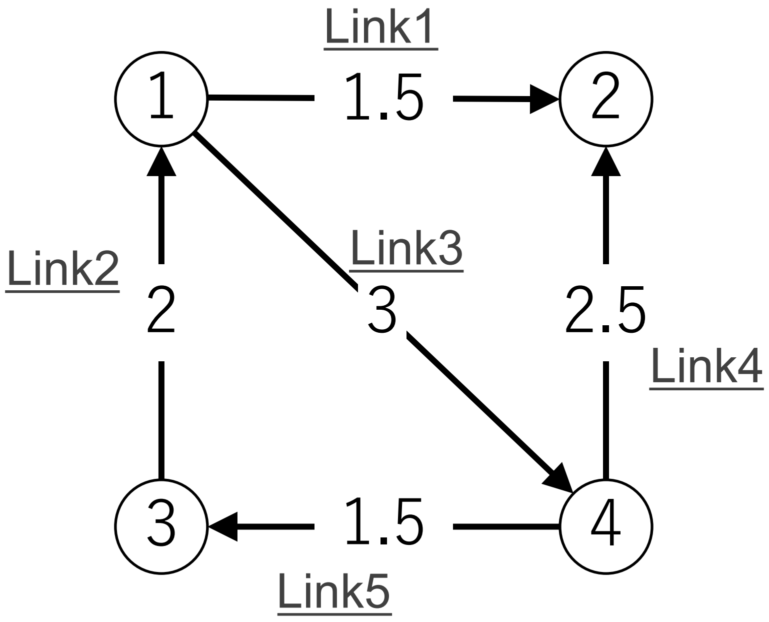
Type of network

- Directed network 有向グラフ
    - Each link has a direction. Each link connects from tail node to head node.
    各リンクが方向を持つ
- Weighted network 重み付きグラフ
    - Each link have element as weight.
    各リンクが重みを持つ

Directed and weighted adjacency matrix $\mathbf{A}$

$$\begin{array}{cc} & \text{Destination} \\ \text{Origin} & \begin{pmatrix} 0 & 1.5 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 2.5 & 1.5 & 0 \end{pmatrix} \end{array}$$

$$a_{ij} = \begin{cases} w_e \text{ if } e = (i,j) \in \mathbf{E} \\ 0 \ \text{ otherwise} \end{cases}$$

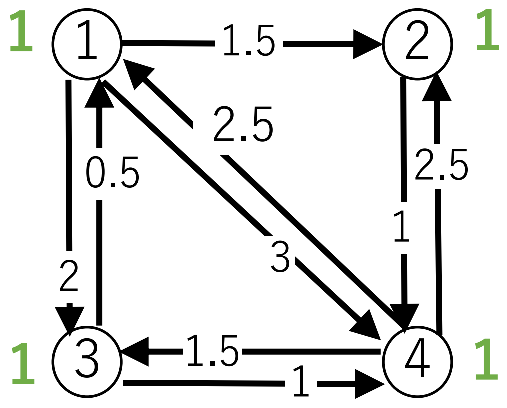


Road Network Evaluation by Eigenvector Centrality

- Set the various feature values as weight for each link to consider the traffic function.
- Evaluate the effect based on each weight setting by weighted eigenvector centrality.

$$\mathbf{Ax} = \lambda\mathbf{x}$$

$$a_{ij} = \begin{cases} \text{weight value of the link from node } i \text{ to node } j \\ 0 \; otherwise \end{cases}$$



$$\mathbf{A} = \begin{pmatrix} 0 & 1.5 & 2 & 3 \\ 0 & 0 & 0 & 1 \\ 0.5 & 0 & 0 & 1 \\ 2.5 & 2.5 & 1.5 & 0 \end{pmatrix}$$

- Iterate to normalise for network size to avoid divergence.
- This iteration converges to the largest eigenvalue. The eigenvector corresponding largest eigenvalue should be all positive. (Perron-Frobenius theorem)

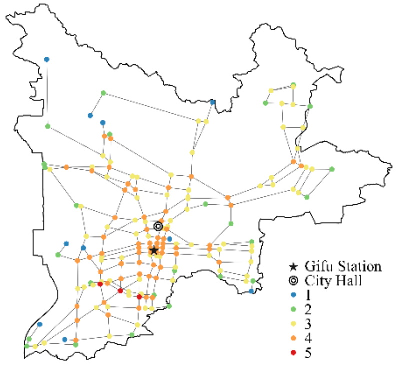
Comparison with other centrality measures

Target centrality measures

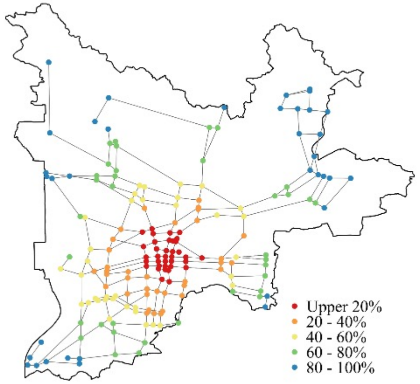
Centrality measure	Reference	Formulation	Definition
Degree Centrality	Proctor & Loomis (1951)	$x_i = \sum_j^n a_{ij}$	The number of links connected to the node.
Closeness Centrality	Beauchamp (1965)	$x_i = \frac{n}{\sum_j d_{ij}}$	The mean distance from a node to other nodes using the shortest path through a network between two nodes.
Eigenvector Centrality	Bonacich (1972)	$x_i = \sum_j A_{ij} x_j$	A node's importance in a network is increased by having connections to other nodes that are themselves important.
Betweenness Centrality	Freeman (1977)	$x_i = \sum_{st} \frac{n_{st}^i}{g_{st}}$	The extent to which a node lies on the shortest paths between other nodes.

Comparison with other centrality measures

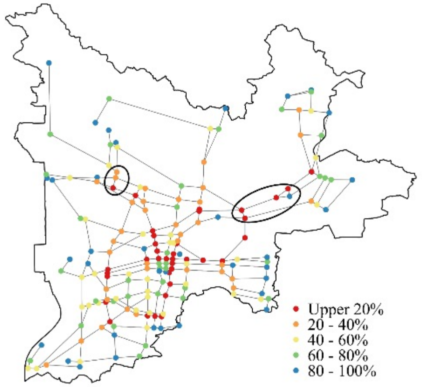
A small-scale road network in Gifu City



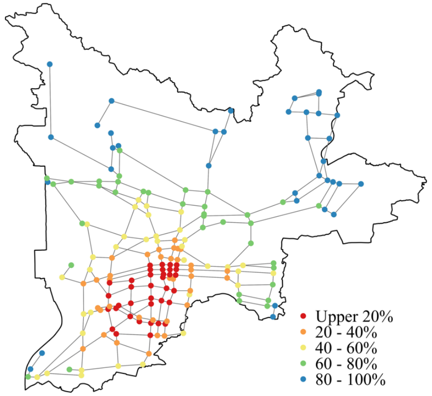
Degree Centrality



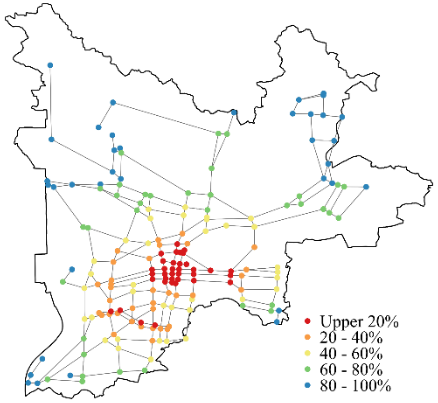
Closeness Centrality



Betweenness Centrality



Eigenvector Centrality



Eigenvector Centrality

Capacity weighted

- The similarity between CC and EC.

  CCとECの相関が強い
- DC is significantly effected by bypass road.

  DCはバイパス道路の影響が強い
- EC includes the characteristics of both DC and CC.

  ECはDCとCCの両方の特性を持つ

Table. Spearman's rank correlation

	Degree Centrality	Closeness Centrality	Betweenness Centrality	Eigenvector Centrality	Eigenvector Centrality Capacity weighted
Degree Centrality	1.0000				
Closeness Centrality	0.5004	1.0000			
Betweenness Centrality	0.5554	0.5132	1.0000		
Eigenvector Centrality	0.5717	0.7679	0.3546	1.0000	
Eigenvector Centrality Capacity weighted	0.4974	0.9263	0.4646	0.8405	1.0000

The similarity suggests the big advantage of eigenvector centrality since the calculation of shortest pass is not needed.

Classification of functional and geographical characteristics of road networks

Weight settings on eigenvector centrality

Classification of Challenges	Weight	Equation	Eigenvector Centrality
The evaluation of road improvement	Capacity	$w_e = C_e$	The magnitude and strength of movement ability on road network. Connectivity considering the ease of link disruption based on the traffic capacity.
	Road area	$w_e = L_e C_e$	Contribution for the supply performance by the road improvements.
Characterised the region on the road network	Speed	$w_e = S_e$	The distribution of road rank connectivity. Connectivity distribution of links with high and low speed limits.
	Capacity	$w_e = C_e$	The magnitude and strength of movement ability on road network. Connectivity considering the ease of link disruption based on the traffic capacity.
	BPR function	$w_e = t_{0i}\left(1 + \alpha P_e^{\beta}\right)$	The distribution of road rank connectivity. Connectivity distribution of links with short and long travel time considering congestion.
	Travel time	$w_e = t_e$	The distribution of road rank connectivity. Connectivity distribution of links with short and long travel time.
	Distance	$w_e = L_e$	The spatial density of network by the connectivity of length on each link.
The usage situation of road network	Congestion rate	$w_e = \frac{V_e}{C_e}$	Concentration and distribution of crowded roads.
	Traffic volume	$w_e = V_e$	Concentration and distribution of traffic volume.

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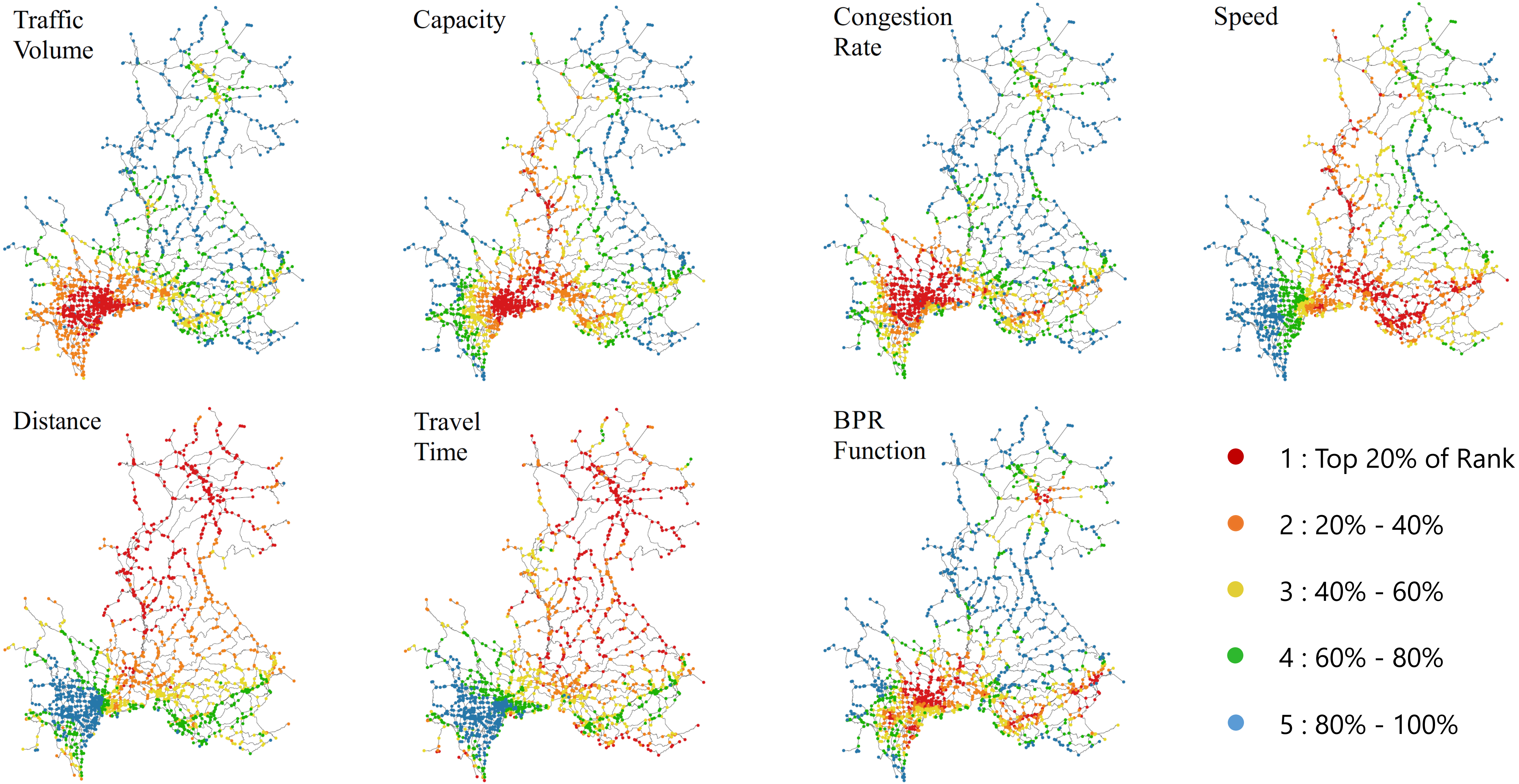
Eigenvector Centrality by Each Traffic Feature Values



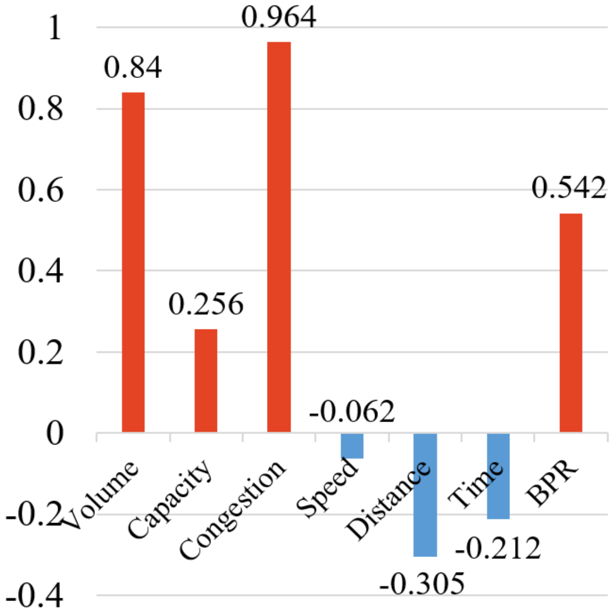
Factor Analysis

To identify the potential common factors.

$$\mathbf{Y} = \mathbf{\Lambda f} + \boldsymbol{\varepsilon}$$

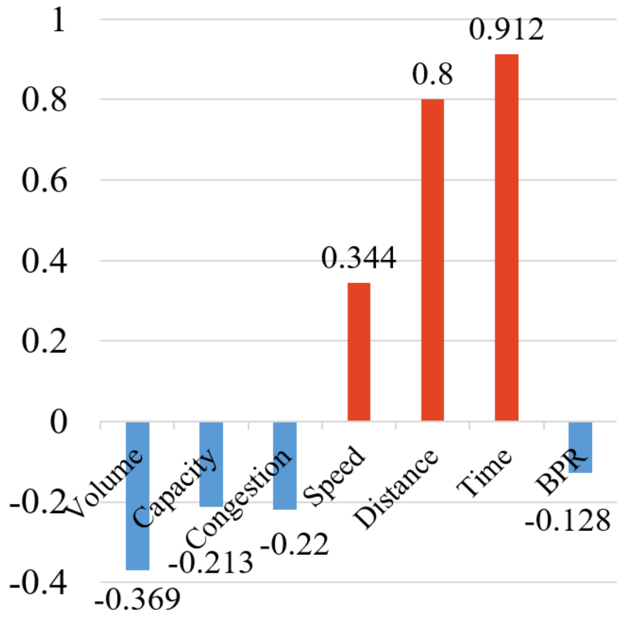
$\mathbf{Y}$: The set of observed variables, $\mathbf{\Lambda}$: The set of factor loadings of each variable and each factor, $\mathbf{f}$: The set of common factors, $\boldsymbol{\varepsilon}$: The set of unique factors

- Three common factors are extracted by factor analysis



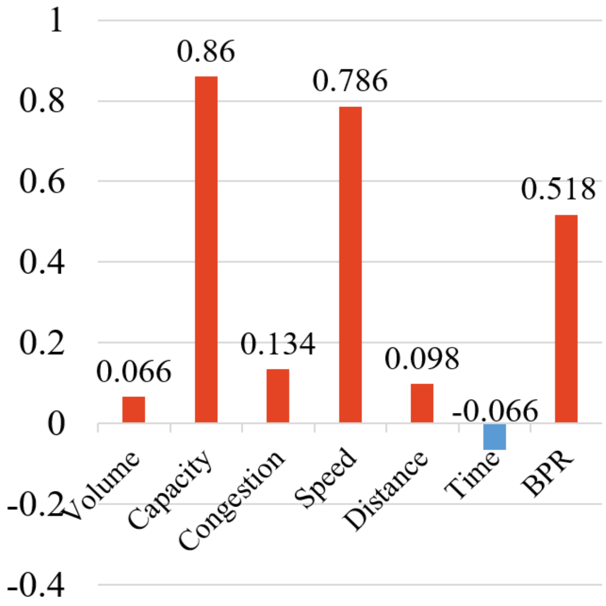
1) First factor loading

Traffic Demand Factor



2) Second factor loading

Road Sparsity Factor



3) Third factor loading

Road Rank Factor

Cluster classification

- Cluster 1【472 : 26%】"Rural part cluster"

Gero City, Ena City, the north part in Shirakawa town and Hida City,

- Cluster 2【372 : 21%】"Western urban part cluster."

Oogaki, Motosu, Ibigawa, Kaizu.

- Cluster 3【140 : 8%】"Northern urban part cluster"

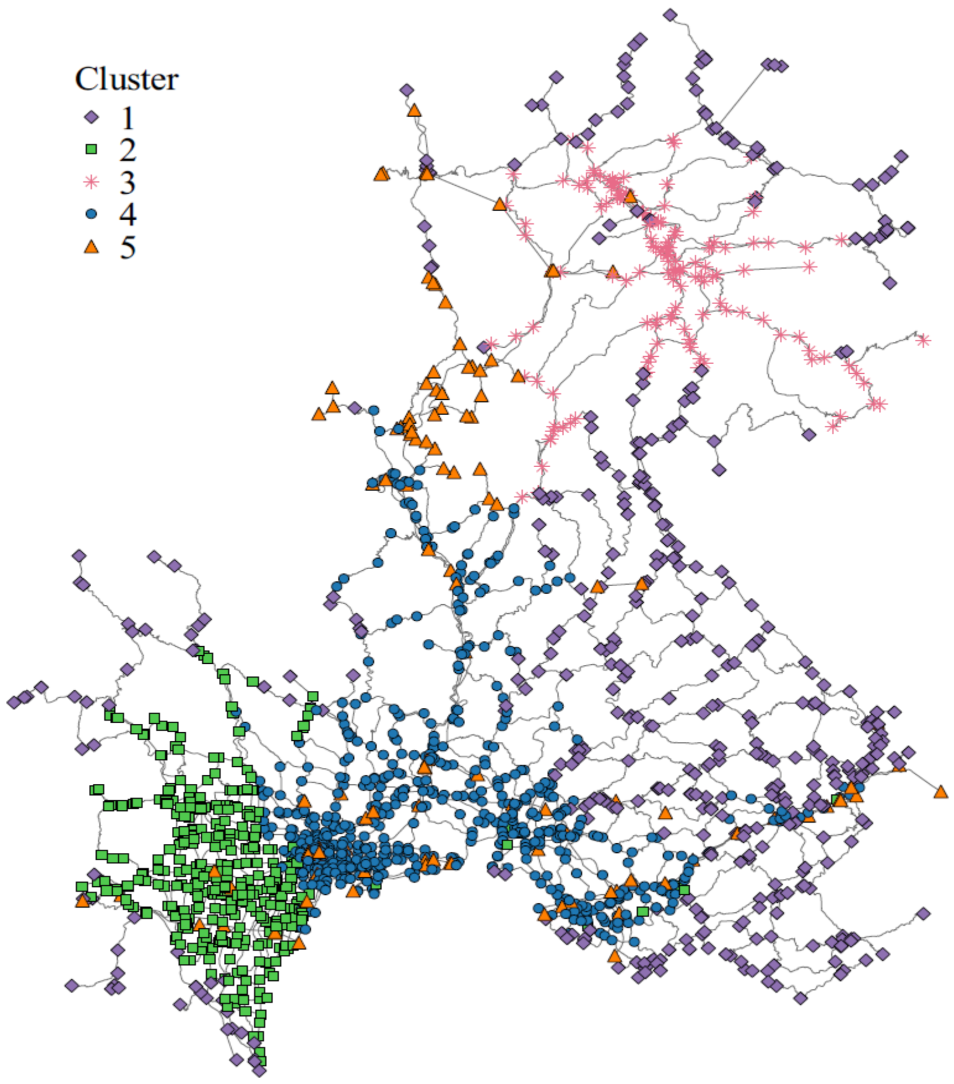
Takayama City

- Cluster 4【678 : 38%】"Central urban part cluster"

Gifu, Kakamigahara, Minokamo, Seki, Gujo

- Cluster 5【121 : 7%】"Expressway cluster"

Nodes are mainly located along the expressway



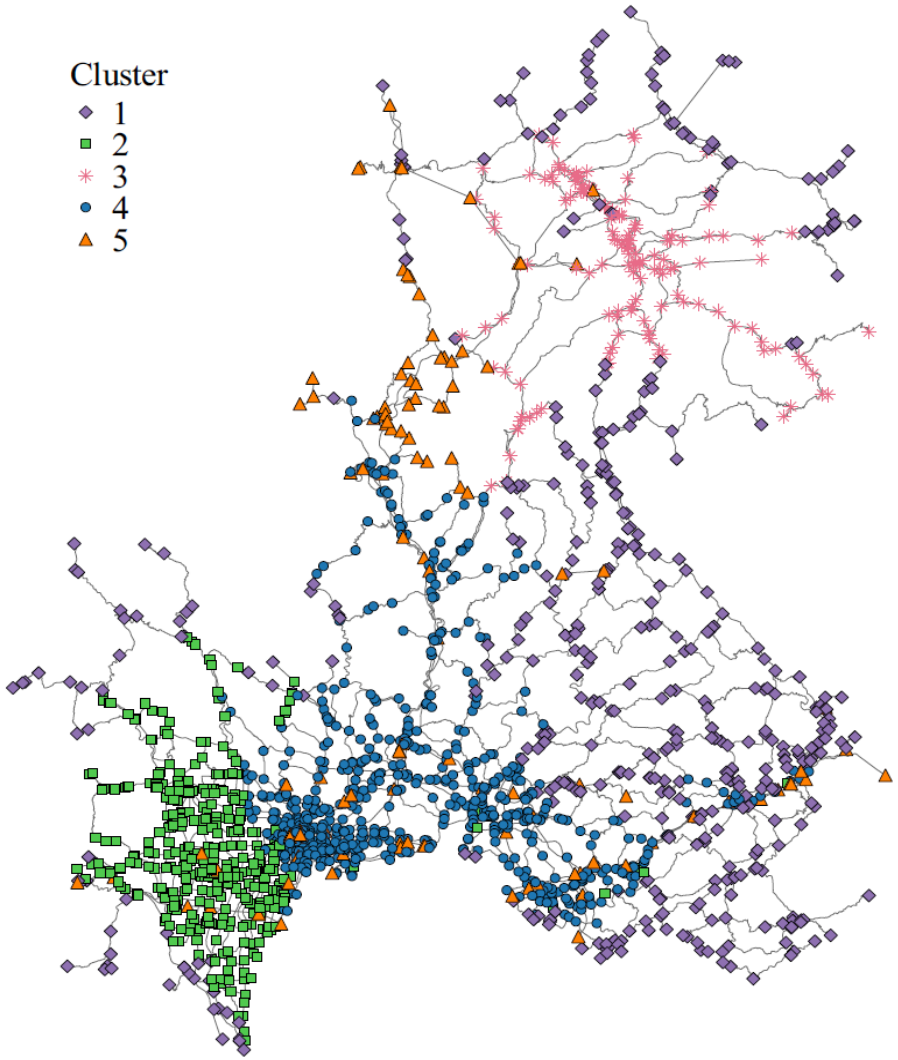
Result of factor analysis by using eigenvector centrality classified the road network by functional and geographical characteristics.

ECに基づく因子分析により，道路ネットワークを機能的・地理的特性によって分類

Cluster classification

Cluster	The number of nodes	Percentage	Average factor score 1 “Traffic demand”	Average factor score 2 “Road sparsity”	Average factor score 3 “Road rank”
1	472	26%	-0.376	0.348	-0.830
2	372	21%	0.378	-1.267	-0.729
3	140	8%	0.247	2.252	-0.541
4	678	38%	0.479	0.060	0.861
5	121	7%	-2.665	-0.402	1.281

Average factor score in each cluster



- Cluster 2【372 : 21%】“Western urban part cluster.”

Oogaki, Motosu, Ibigawa, Kaizu.

Traffic demand : High demand

Road sparsity : Dense

Road rank : Low

Large capacity roads are insufficient despite high demand and dese networks. For road improvement, capacity expansion of existing links is effective.

需要が高いにも関わらず大容量の道路が不十分，ネットワーク構造は発達している．既存リンクの容量拡大が有効.

Relationship between road improvement and usage in the long term

Weight settings on eigenvector centrality

Classification of Challenges	Weight	Equation	Eigenvector Centrality
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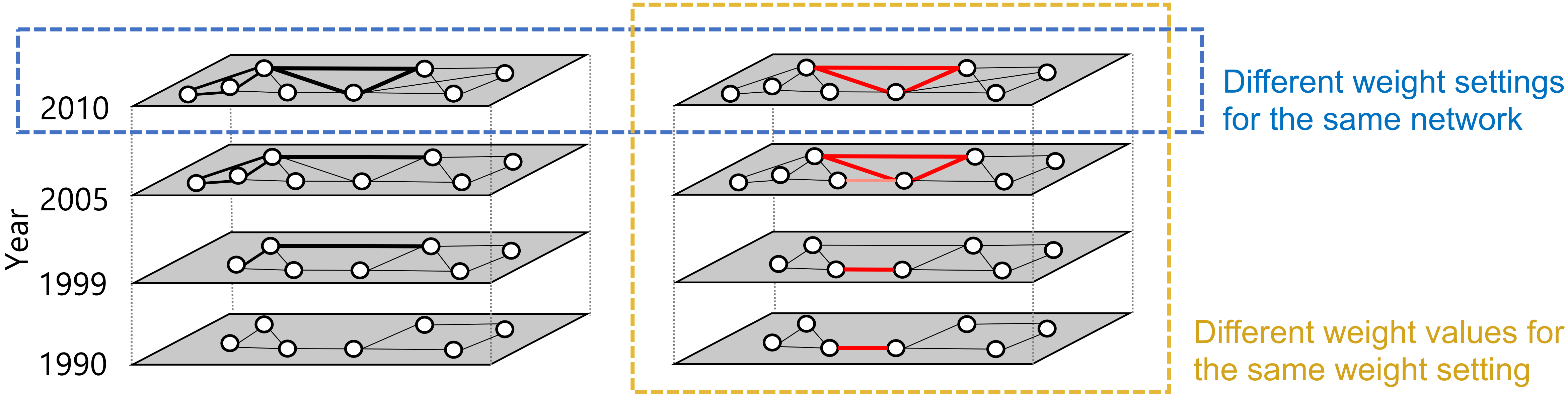
Multiple-year data

- Target area : Gifu Prefecture, Japan
  - ➢ 10,620 ㎢ in size and included mountainous and urban areas.
- Target years : 1990, 1999, 2005, 2010
  - ➢ The road networks have been improved year-on-year

The number of nodes and links in road networks by year

Year	1990	1999	2005	2010
Node	1727	1770	1791	1793
Link (Ordinary roads)	4494	4618	4717	4723
Link (Expressway roads)	35	52	69	73
Total lengths (km)	9000	9495	9798	9857



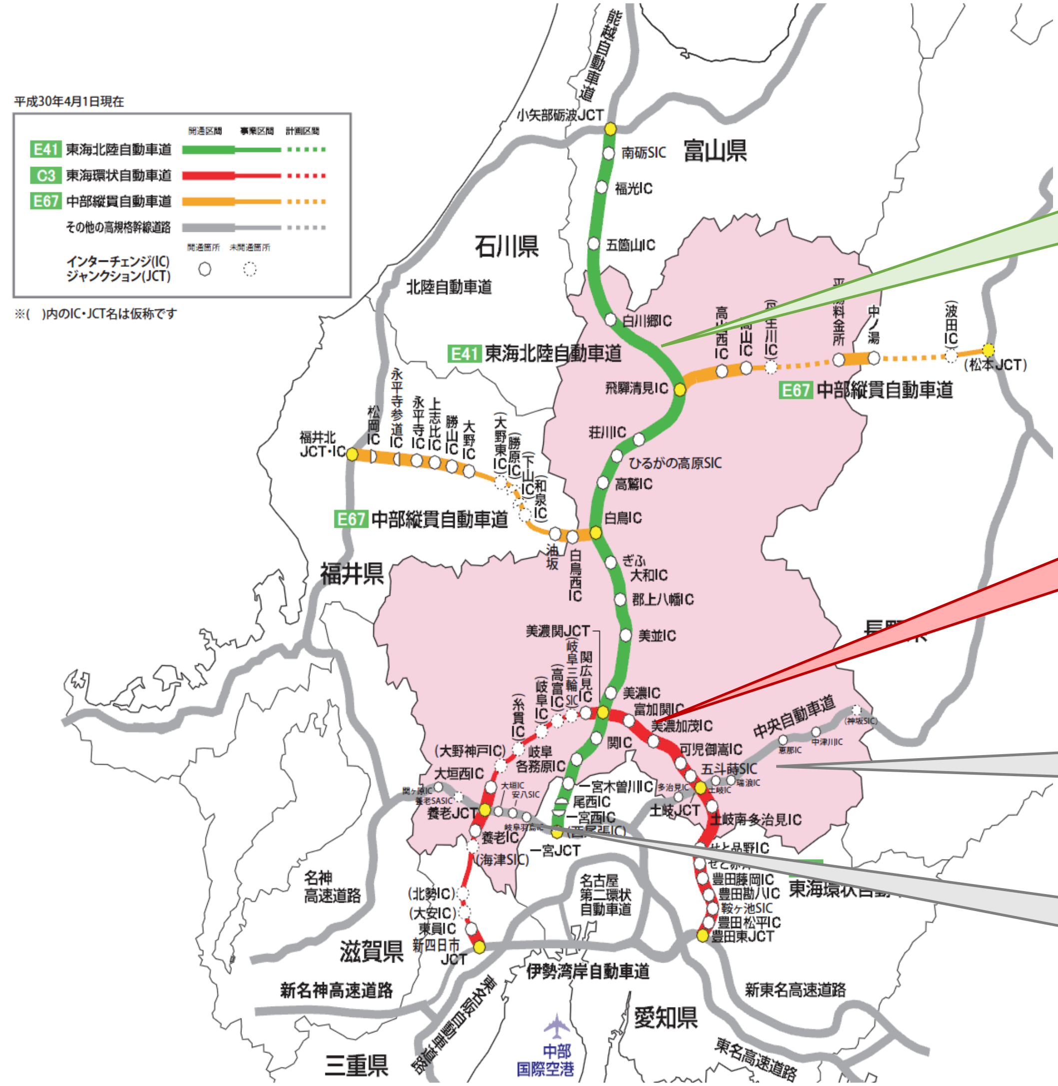
Supply Side : $w_e = L_e C_e$

Demand Side : $w_e = V_e$

Concept figure

The change of road network in Gifu

Improvement of road network



Tokai Hokuriku Expressway
Partial opened in 1986
All opened in 2008
Under construction for 4 lanes

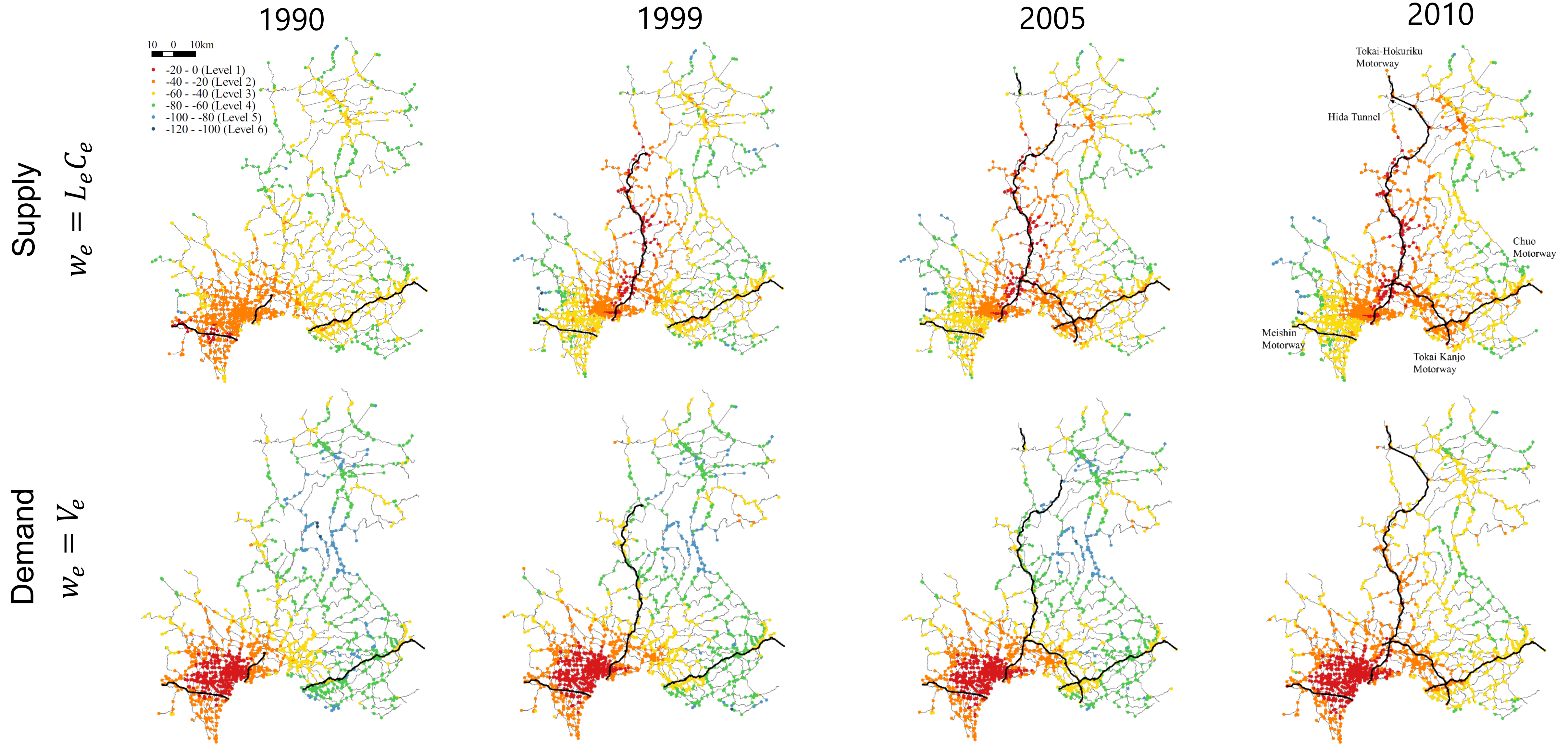
Tokai Ring Expressway
East side opened in 2005
Under constriction for west side

Chuo Expressway
All roads have been opened

Meishin Expressway
All roads have been opened

Impact of road improvements on supply and demand

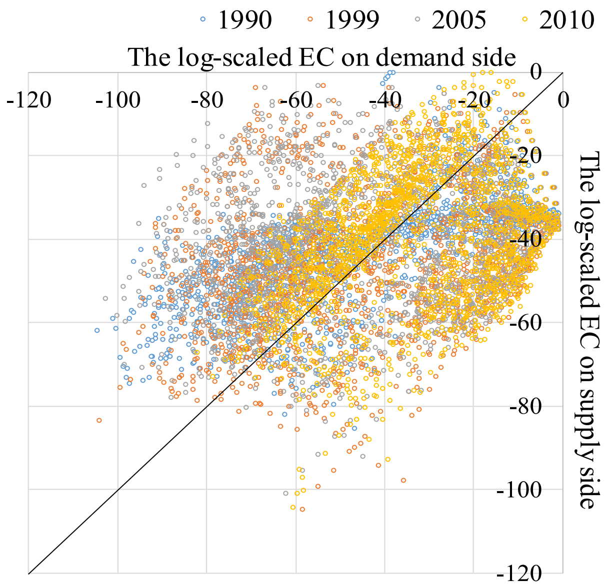
The change of EC based on supply and demand



Impact of road improvements on supply and demand

Scatter plots of logarithm eigenvector centrality values on supply side and demand side for each year

各年度の供給側と需要側の対数固有ベクトル中心性の散布図



The slope of the linear approximation line through the origin

Year	Slope	R^2
1990	0.808	0.880
1999	0.834	0.755
2005	0.737	0.712
2010	1.002	0.774

In 1990-2005, many nodes have lower demand side evaluations than supply side evaluations

2005年までは需要側の評価が供給側の評価より低いノードが多い

Connectivity on road performance have improved by road development on the supply side, however impacts on the demand side are not as large as the supply side?

道路整備により供給側が示す道路性能の接続性は向上したが，需要側への影響はそれほど大きくないのか？

Correlation of supply and demand

Correlation coefficients of both weights

	Year	Supply 1990	Supply 1999	Supply 2005	Supply 2010
Demand	1990	0.720	0.300	0.140	0.131
Demand	1999	0.715	0.289	0.154	0.140
Demand	2005	0.728	0.289	0.157	0.143
Demand	2010	$\boxed{0.764}$	0.478	0.361	0.349

The correlation between supply in 1990 and demand in 2010 is high, even though the EC is calculated with different weights and years.

年度が離れているのにも関わらず，1990年供給量と2010年需要量の相関が最も高い

- Road network is improved because of the increase in demand

or

- Demand increased by the results of road improvements.

Correlation of supply and demand

Correlation coefficients of both weights

		Supply			
	Year	1990	1999	2005	2010
Demand	1990	0.720	0.300	0.140	0.131
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The correlation between supply in 1990 and demand in 2010 is high, even though the EC is calculated with different weights and years.

- Road network is improved because of the increase in demand

There should be the correlation between past demand and future supply.

過去の需要に対して供給の年数が進むほど相関が強くなるはず

The correlation coefficient continually decrease, this suggestion does not occurred.

相関係数は減少し続けており，仮定と反する

Correlation of supply and demand

Correlation coefficients of both weights

		Supply			
	Year	1990	1999	2005	2010
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The correlation between supply in 1990 and demand in 2010 is high, even though the EC is calculated with different weights and years.

- Demand increased by the results of road improvements.

There should be the correlation between past supply and demand subsequence year.

過去の供給に対して需要の年数が進むほど相関が強くなるはず

The correlation coefficient continually increase.

相関係数は経年的に増加

<u>There is a lagged effect of road investment.</u>

Demand-side connectivity increases as supply-side connectivity becomes better by road improvements. 道路整備により供給側の接続性が向上したのちに，需要側の影響の拡がりが追従する

Conclusions

- Eigenvector centrality analysis showed that it is possible to evaluate a road network from different perspectives by using some weight settings.
- Factor analysis has been used to identify important factors (traffic demand, road density and road rank) for characterizing the road network.
- The supply side and demand side evaluations by topological approach showed differences in impacts of road network improvement.
- The evaluation of changes over a 20-year period using real road networks suggests that there may be a time lag in the impacts of road network improvement on demand side.

Network inherent structure

The eigenvector centrality measure is relative evaluation index within a network, it is difficult to evaluate the impact of network topology changes due to disasters, urban development, etc. 固有ベクトル中心性指標はネットワーク内の相対的な評価指標であり、形状変化の影響を直接的に評価することはできない

1. Absolute evaluation by the leading eigenvalue, an index uniquely determined for the entire network ネットワーク全体に対して一意に決まる指標である最大固有値による絶対評価
   - To obtain the unique leading eigenvalue in directed graph, the graph must be strongly connected. 有向グラフにおいて唯一に求めるには強連結なグラフでないといけない
   - The leading eigenvalue is evaluated higher when there are extremely strong parts. So, the evaluation by the leading eigenvalue has weak affinity with homogeneity and fairness. 最大固有値は極端に強く接続する部分があればあるほど高い値を示し，均質性や公平性とは異なる視点となる
2. Evaluate the impact of missing parts by the amount of the eigenvector centrality change from a criteria value determined by the network size

   ネットワークサイズによって決まる基準値からの固有ベクトル中心性変化量によって、形状や特徴の変化がもたらす影響を評価する

Thank you !