EPFL

September 12, 2024

Modeling and optimization for emerging mobility

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EPFL

Our lab

Members

Head

Postdoc

PhD

Visiting

Lab for human-oriented mobility eco-system

At HOMES, we develop human-centric solutions to emerging mobility challenges.

Our lab

HOMES

Topics

Shared mobility

- ride-hailing & ride-pooling
- ride-sharing
- micro-mobility

+ integration into mass transit

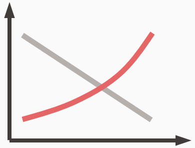
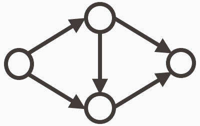
Traffic demand management

- network design
- incentives design

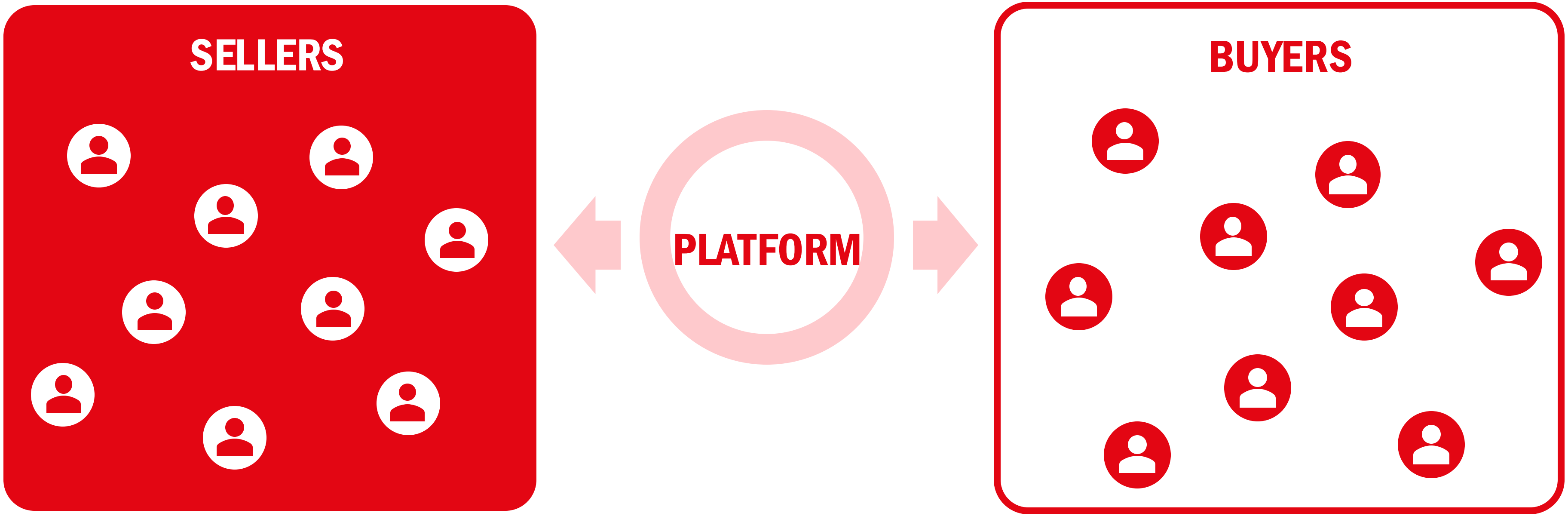
Methodologies

- network modeling
- game theory
- min F(x) optimization
- data-driven & learning

Outline

	Modeling	Optimization	Application
 Aggregate	▪ two-sided market ▪ platform competition	▪ fixed-point iteration ▪ MPEC with fixed point	▪ ride-hailing ▪ micromobility ▪ meal delivery
 Network	▪ mixed traffic ▪ multi-modal travel ▪ traffic management	▪ traffic assignment ▪ MPEC with VI	▪ AV routing ▪ MaaS

What is a two-sided market?



Two-sided markets in transport. systems

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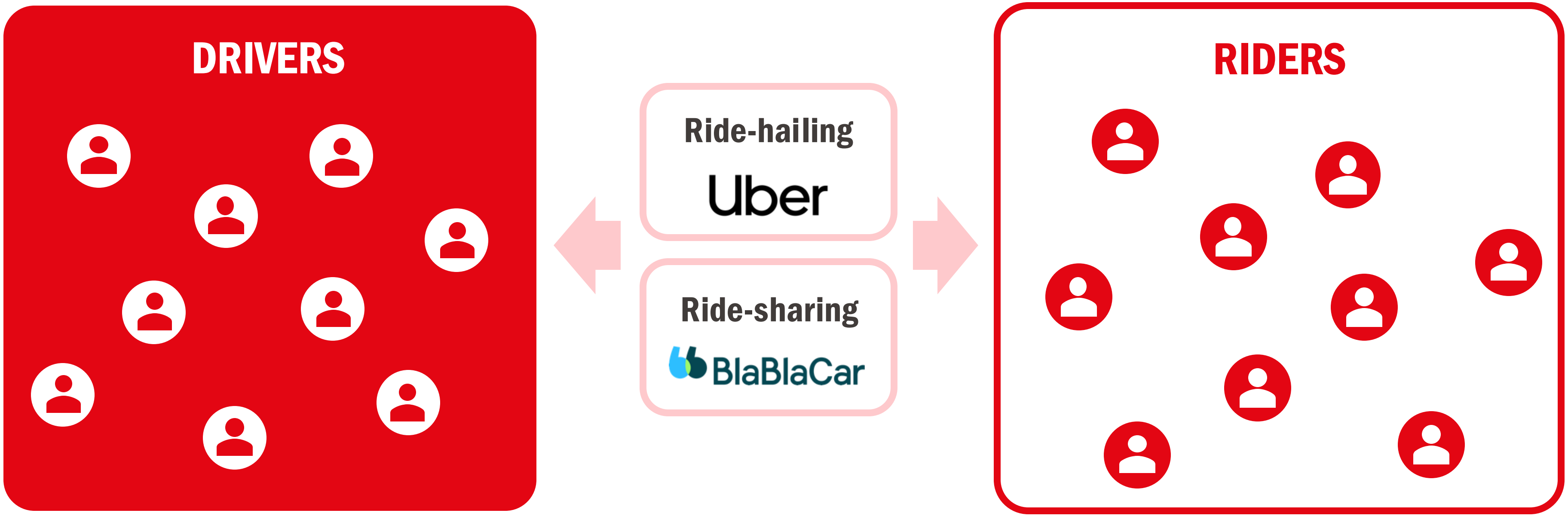
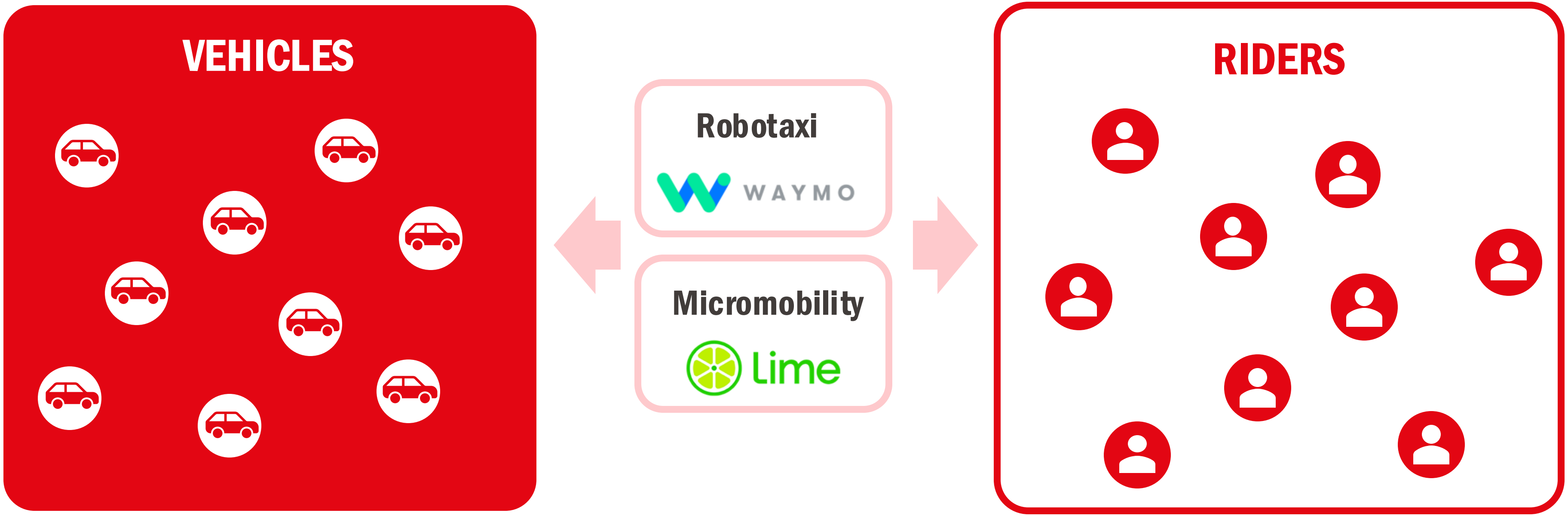
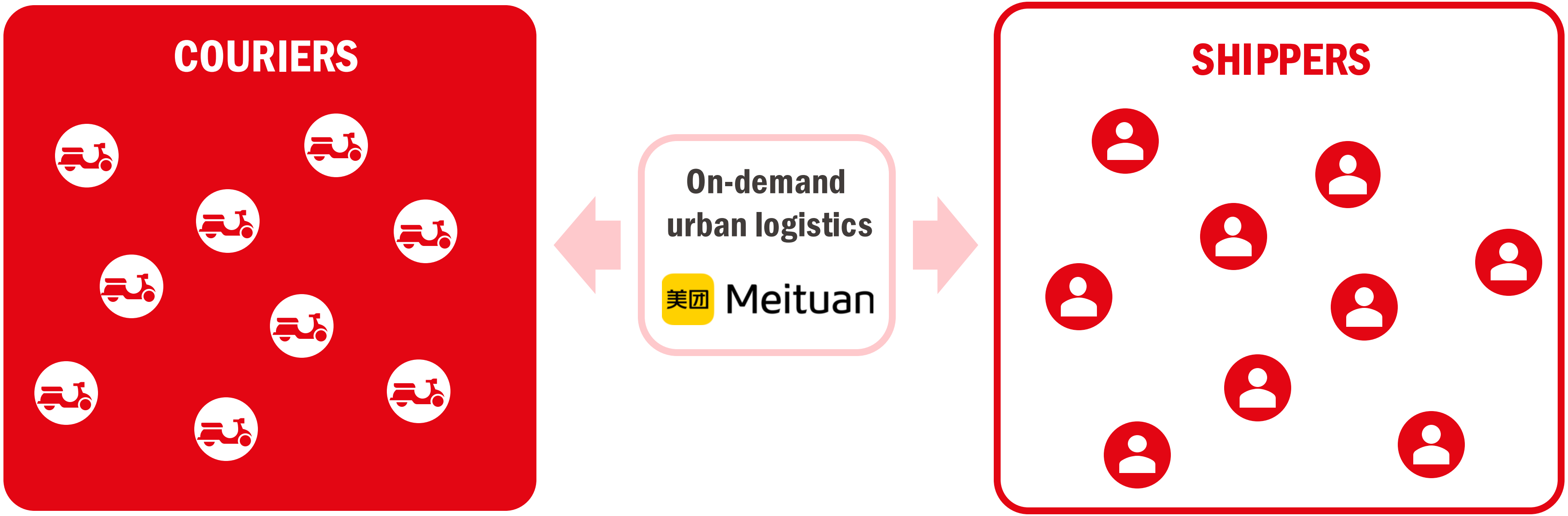


Two-sided markets in transp. systems

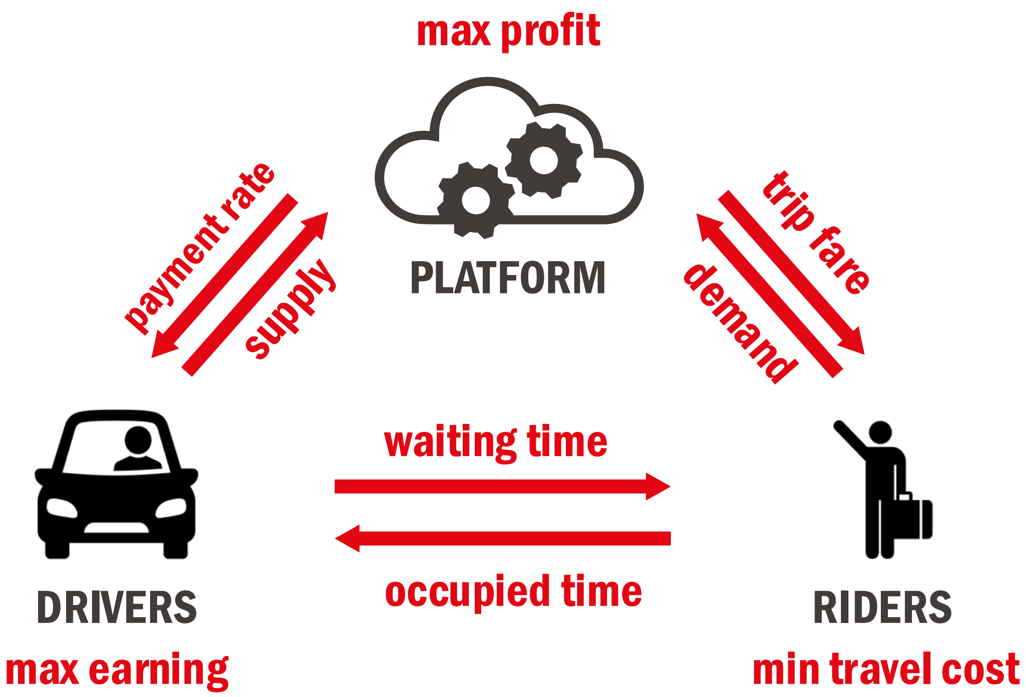


Two-sided markets in transp. systems



Stakeholders

- Ride-hailing as an example



Stakeholders

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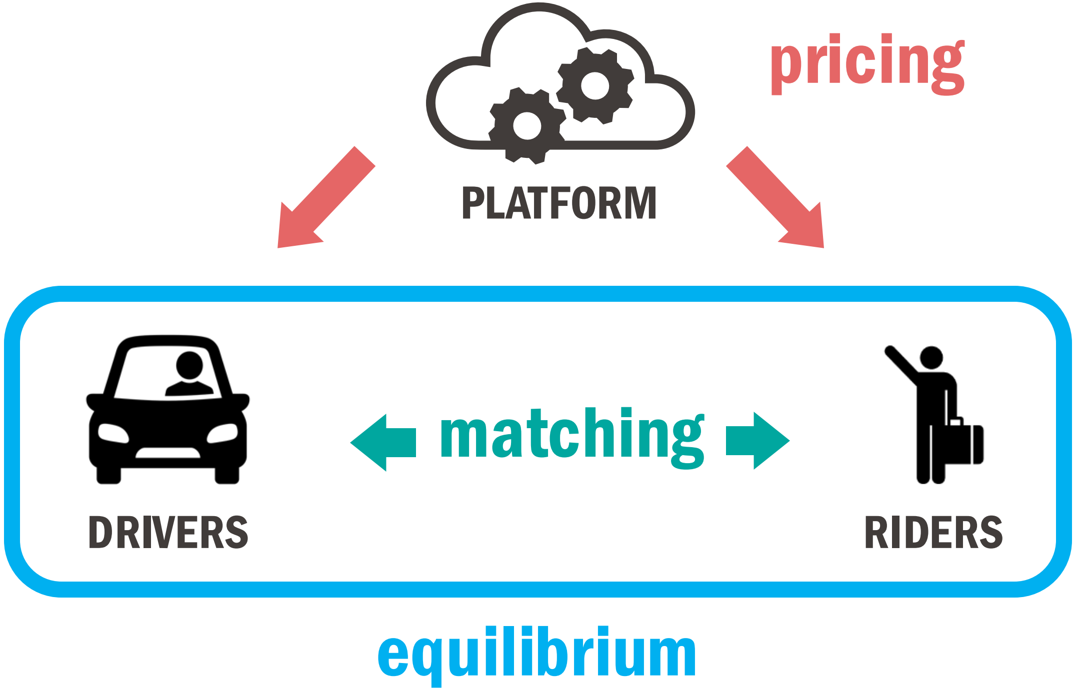

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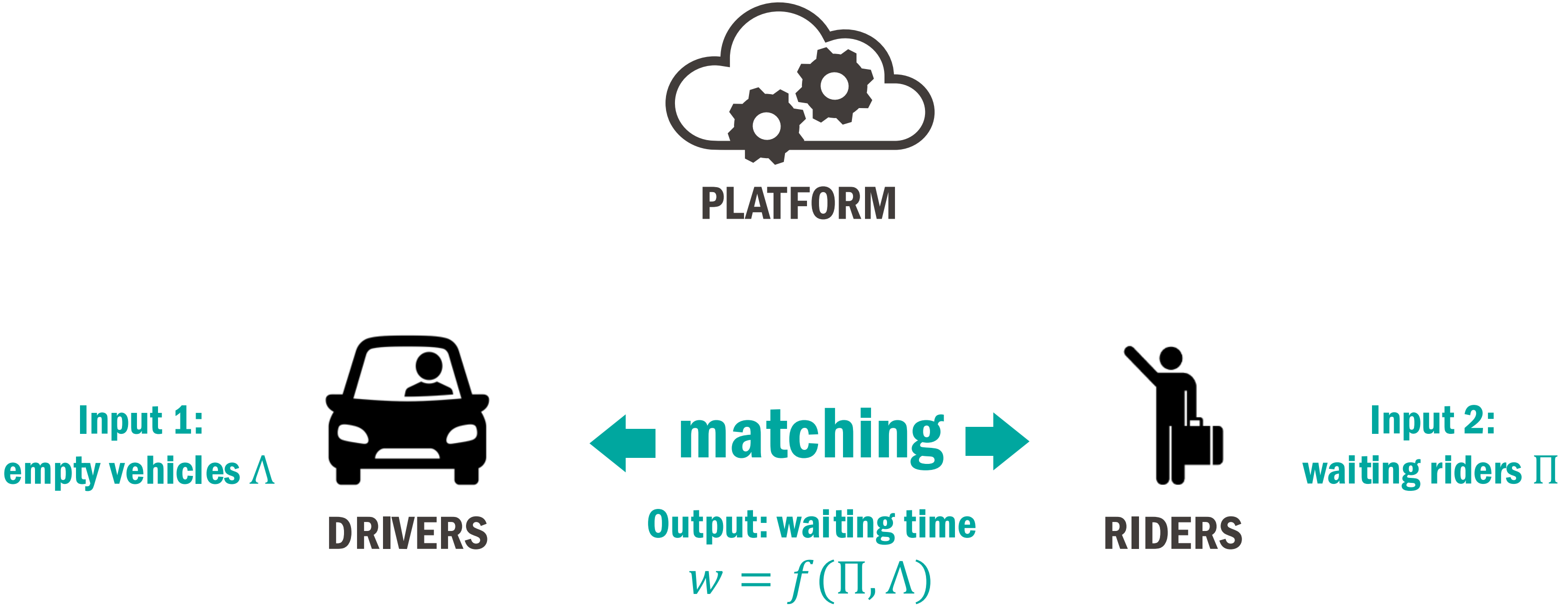
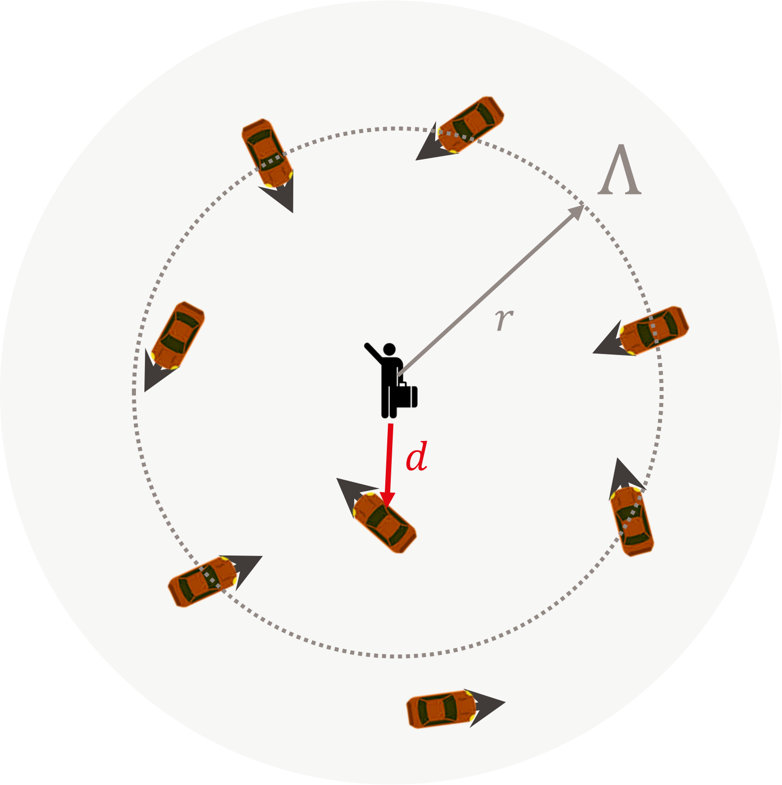
Stakeholders

- Ride-hailing as an example



Matching

- Radio-dispatch: the simplest case



Assume empty vehicles distributed uniformly over space at density Λ

When rider arrives, # empty vehicles within a distance r follows a spatial Poisson distribution[1] $N(r)$

Suppose rider is picked up by the closest empty vehicle at distance d

$$\Pr\{d = r\} = 1 - \Pr\{N(r) = 0\} = 1 - \exp\left(-\int_0^r 2\pi\Lambda x \, \mathrm{d}x\right)$$

Given vehicle speed v and network detour ratio δ, the rider waiting time is $w = \delta d / v$ w.p.

$$\Pr\{w = t\} = 1 - \exp\left(-\pi\Lambda\left(\frac{vt}{\delta}\right)^2\right),$$

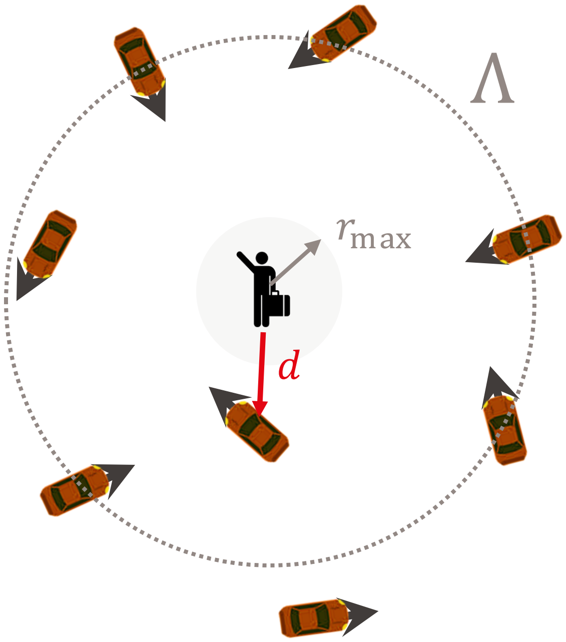
and expectation

$$\mathbb{E}[w] = \frac{\delta}{2v\sqrt{\Lambda}}$$

1 Larson and Odoni. Urban operations research. 1981.

Matching

- Street-hailing: limited matching radius



Street-hailing riders hail empty vehicles on streets, thus the matching radius $r_{\max}$ is constrained by visual range and blockage

Only a small fraction $p(r)$ of empty vehicles would finally enter the matching area defined by $r_{\max}$, thus the pickup vehicle is at distance d w.p.

$$\Pr\{d=r\} = 1 - \exp\left(-\int_0^r 2\pi\Lambda p(x) x \, \mathrm{d}x\right)$$

With some approximations[1], the distribution of rider waiting time is derived as

$$\Pr\{w=t\} = 1 - \exp\left(-\sigma r_{\max}\Lambda\left(\frac{vt}{\delta}\right)\right),$$

with expectation

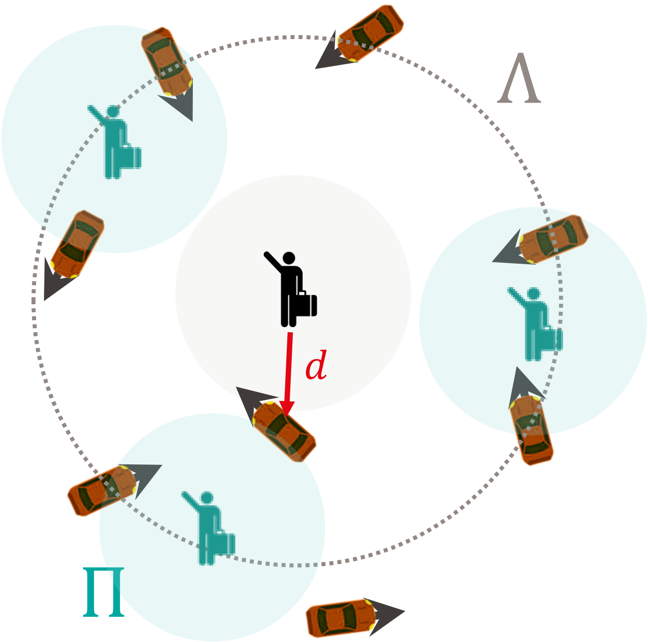
$$\mathbb{E}[w] = \frac{\delta}{\sigma r_{\max} v \Lambda},$$

where σ is a parameter that describes search behaviors

1 Zhang et al. An efficiency paradox of uberization. 2019.

Matching

- E-hailing: potential passenger competition



E-hailing (e.g., Uber) often matches a large number of riders and vehicles in real-time, which induces a competition for empty vehicles among waiting riders

Suppose empty vehicles are evenly allocated to riders, then the pickup distance d follows

$$\Pr\{d = r\} = 1 - \exp\left(-\int_0^r \frac{2\pi\Lambda}{\Pi} x \,\mathrm{d}x\right)$$

The distribution of rider waiting time is then derived as

$$\Pr\{w = t\} = 1 - \exp\left(-\frac{\pi\Lambda}{\Pi}\left(\frac{vt}{\delta}\right)^2\right),$$

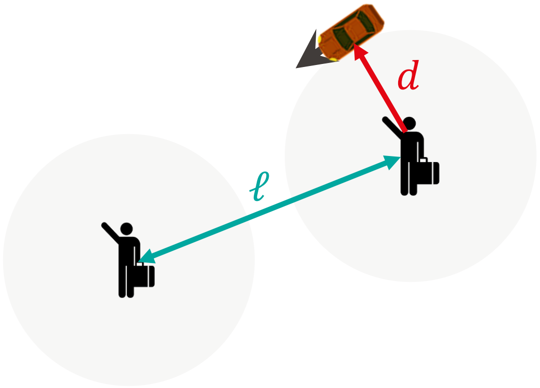
with expectation

$$\mathbb{E}[w] = \frac{\delta}{2v}\sqrt{\frac{\Pi}{\Lambda}}$$

1 Zhang et al. An efficiency paradox of uberization. 2019.

Matching

- Ride-pooling: share ride with another passenger



Ride-pooling (e.g., UberPool) pairs riders and matches them with empty vehicles in real-time

Suppose rider is paired with the closest unmatched rider at distance ℓ and then matched to the closest empty vehicle at distance d

The matched vehicle first picks up the closer rider with time w and then the other with time Δ

The matching area of pooling is expanded thanks to a larger ℓ, whereas a large ℓ leads to a larger Δ[1]

$$\mathbb{E}[w] = \frac{\delta}{2\sigma}\sqrt{\frac{\Pi}{\Lambda}\left(\frac{m + 4\Pi}{2m + 4\Pi}\right)}$$

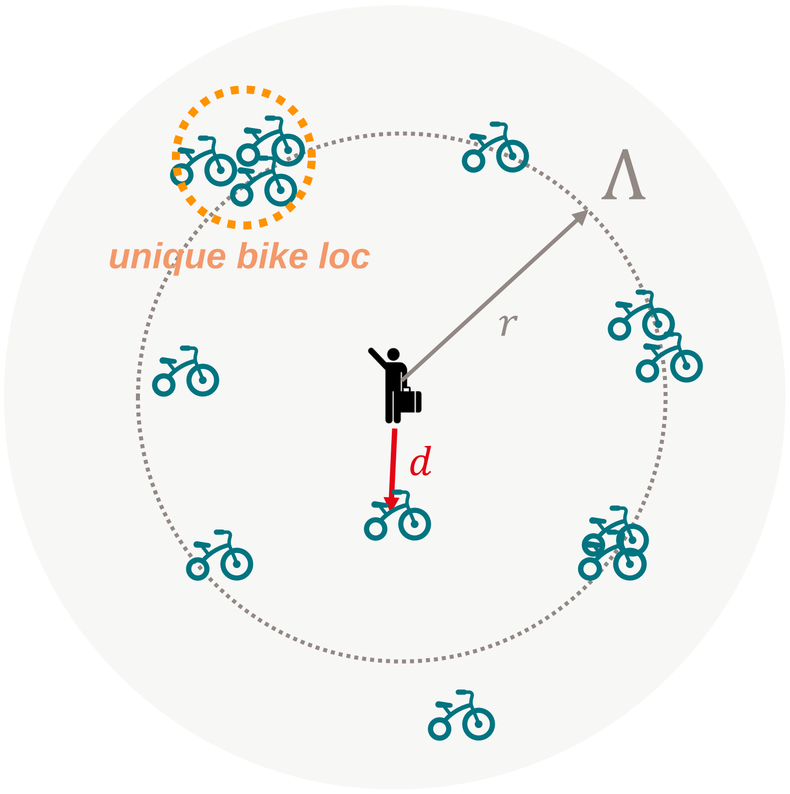
$$\mathbb{E}[\Delta] = \frac{\delta}{2v\sqrt{\Pi}}$$

where m is a parameter for approximation

1 Zhang and Nie. To pool or not to pool: Equilibrium, pricing and regulation. 2021.

Matching

- Bike-sharing: access to idle bikes



Access time in dockless micromobility (e.g., bike-sharing) can be estimated in a similar way as ride-hailing

The key difference is that idle bike density Λ is computed by unique bike parking locations $\bar{n}$ instead of idle bikes n, due to clustering effect[1]

$$\bar{n} = L(n) \leq n$$

Let A be the service region, then the expected access time a is given by

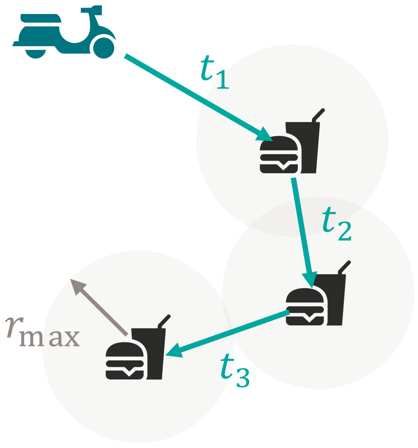
$$\mathbb{E}[a] = \frac{\delta}{2v}\sqrt{\frac{\bar{n}}{A}}$$

where v is the walking speed

1 Zheng et al. How many are too many? Analyzing dockless bikesharing systems with a parsimonious model. 2024.

Matching

- Meal delivery: bundle multiple orders



When demand is high, meal delivery platforms (e.g., Meituan) often group multiple orders with close pickup and delivery locations into bundles and dispatch bundles to idle couriers

The bundling and pickup process is similar to ride-pooling with multiple riders, thus the same matching model can be applied[1]

$$\mathbb{E}[t_1] = \frac{\delta}{2v}\sqrt{\frac{\kappa(\Pi)}{\Lambda}},$$

where $\kappa(\Pi)$ captures the competition effect of bundled orders,

$$\mathbb{E}[t_n] = \frac{\delta}{2v\sqrt{\varphi(\Pi)}}\left[1 - \frac{1}{P}\left(1 - \frac{1}{2r_{\max}\sqrt{\varphi(\Pi)}}\right)\right]$$

where $\varphi(\Pi)$ describes orders that can be grouped into bundles and P is the probability of adding a new order into the current bundle

$$P = 1 - \exp(-\pi\varphi(\Pi)r_{\max}^2),$$

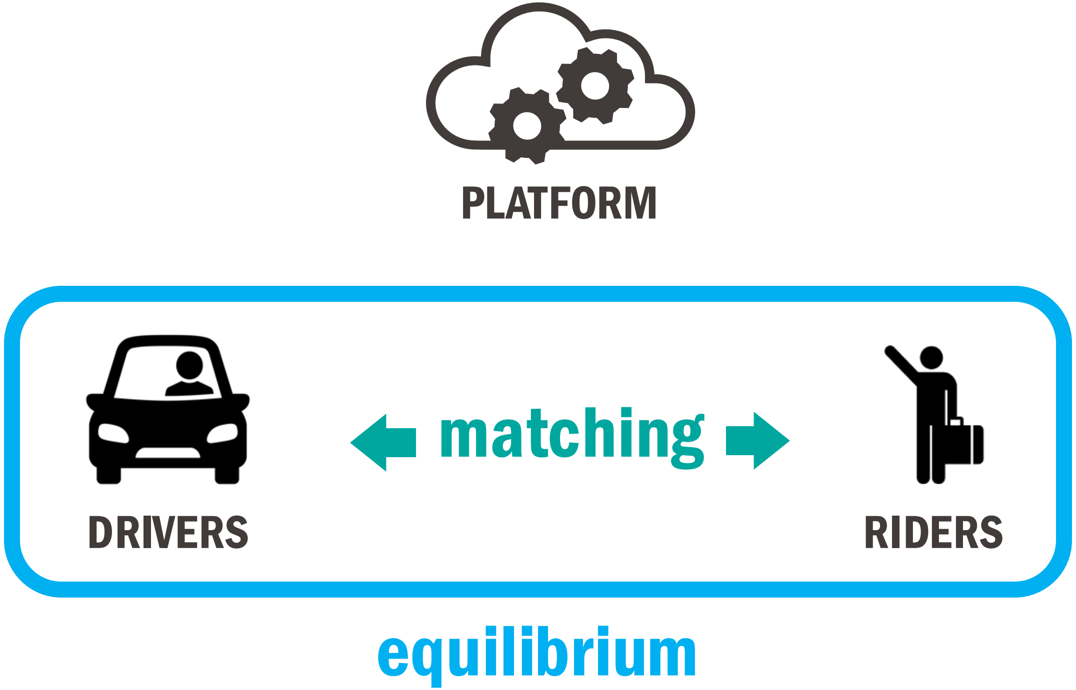
1 Ye et al. Modeling and managing an on-demand meal delivery system with order bundling. 2024.

Summary

- Omit detailed matching but capture key relationship between inputs and outputs
- Describe the physical interactions in various two-sided markets
- Lay a foundation for market equilibrium and operations management

Questions?

Equilibrium



Equilibrium

- Incentives of demand and supply

RIDERS

Demand function

$$Q = D(f, w)$$

min travel cost

$$u = \sum_m P_m[f_m + v(w_m + \tau_m)]$$

- m travel mode
- P_m choice probability
- f_m trip fare
- v value of time
- w_m waiting/access time
- τ_m in-vehicle time

DRIVERS

Supply function

$$N = S(e)$$

max earning

$$e = \sum_k P_k \, e_k$$

- k job opportunity
- P_k choice probability
- e_k earning rate

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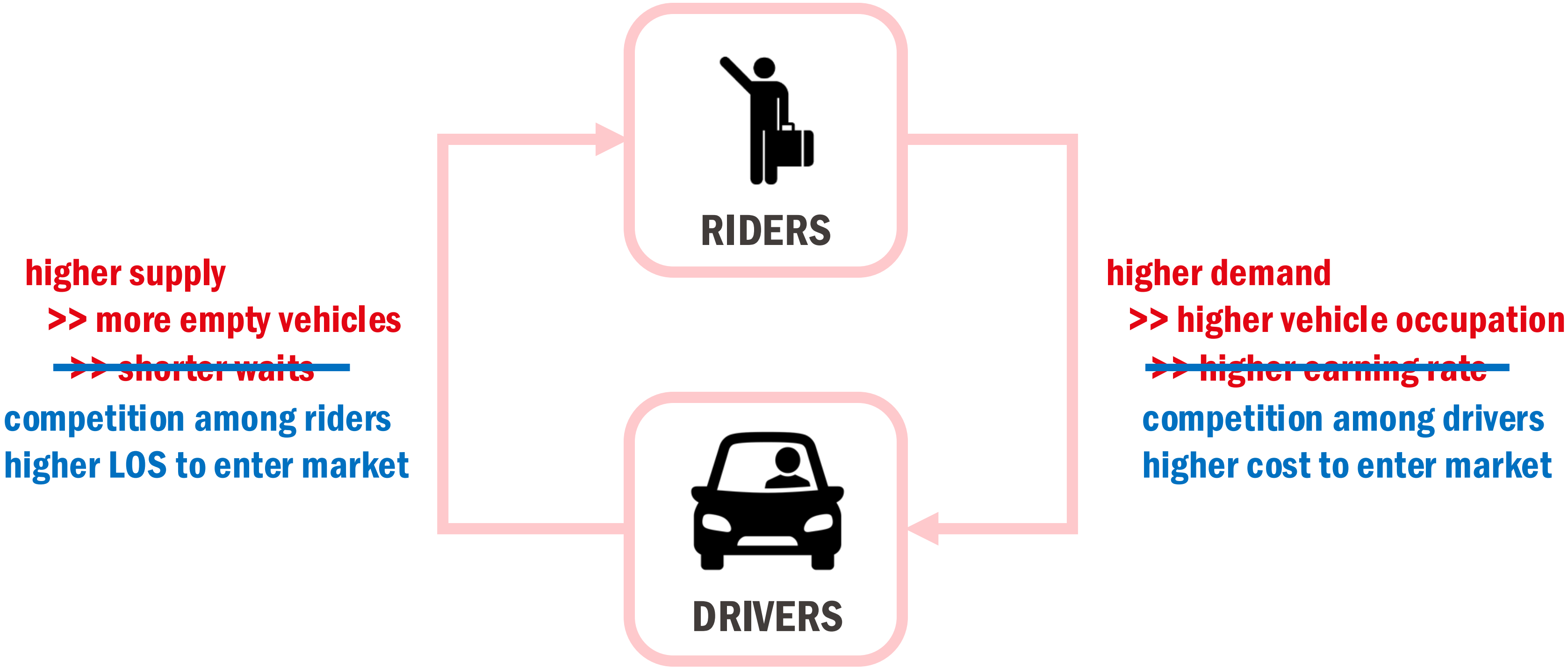
Equilibrium

- Market equilibrium as a fixed point



Equilibrium

- Market equilibrium as a fixed point

Demand function:	$Q = D(f, w)$
Level of service :	$w = W(\Pi, \Lambda)$
Rider matching queue:	$\Pi = Qw$
Supply function:	$N = S(e)$
Earning rate:	$e = \frac{\eta Q \tau}{N}$
Fleet conservation:	$N = \Lambda + Q(w + \tau)$

Platform's decision:

- f: fare per trip
- η: payment per unit occupied time

Equilibrium

- Market equilibrium as a fixed point

Demand function:	~~$Q = D(f, w)$~~
Level of service :	$w = W(\Pi, \Lambda)$
Rider matching queue:	$\Pi = D(f, w)w$
Supply function:	$N = S(e)$
Earning rate:	$e = \frac{\eta\tau}{N} D(f, w)$
Fleet conservation:	$N = \Lambda + D(f, w)(w + \tau)$

Equilibrium

- Market equilibrium as a fixed point

Demand function: ~~$Q = D(f, w)$~~

Level of service : $w = W(D(f, w)w, \Lambda)$

Rider matching queue: ~~$\Pi = D(f, w)w$~~

Supply function: $N = S(\frac{\eta\tau}{N} D(f, w))$

Earning rate: ~~$e = \frac{\eta\tau}{N} D(f, w)$~~

Fleet conservation: $N = \Lambda + D(f, w)(w + \tau)$

Equilibrium

- Market equilibrium as a fixed point

Demand function: ~~$Q = D(f, w)$~~

Level of service : $w = W(D(f, w)w, N - D(f, w)(w + \tau))$

Rider matching queue: ~~$\Pi = D(f, w)w$~~

Supply function: $N = S(\frac{\eta\tau}{N} D(f, w))$

Earning rate: ~~$e = \frac{\eta\tau}{N} D(f, w)$~~

Fleet conservation: ~~$N = \Lambda + D(f, w)(w + \tau)$~~

Let $\mathbf{x} = (w, N)$ and $F = (W, S)$, then the market equilibrium is expressed by a fixed point

$$\mathbf{x}^* = F(\mathbf{x}^*)$$

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Equilibrium

- Existence of equilibrium due to fixed-point theorem

Brouwer's fixed point theorem

If a continuous function $F: \Omega \subset \mathbb{R}^n \rightarrow \Omega$ maps a compact and convex set Ω to itself, then there exists $\mathbf{x}^* \in \Omega$ such that $\mathbf{x}^* = F(\mathbf{x}^*)$

Recall the market equilibrium defined before

- $F = (W, S)$ is a continuous mapping on $\mathbb{R}^2$
- Functions W, S can be designed such that both waiting time w and fleet size N are bounded, i.e., $\Omega \coloneqq [\underline{w}, \overline{w}] \times [\underline{N}, \overline{N}]$.
- The feasible set Ω is then compact and convex

** The uniqueness is however not guaranteed without additional property of F, but usually there exists one stable equilibrium*

Equilibrium

- Solve equilibrium by fixed-point iterations
  - Initialize with a feasible solution $\mathbf{x}^0$
  - At each iteration n, update solution by

$$\mathbf{x}^{n+1} = F(\mathbf{x}^n)$$

  - Terminate when $||\mathbf{x}^{n+1} - \mathbf{x}^n|| \leq \varepsilon$ for some gap threshold ε

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Equilibrium

- Solve equilibrium by fixed-point iterations
  - Initialize with a feasible solution $\mathbf{x}^0$
  - At each iteration n, update solution by

    $$\mathbf{x}^{n+1} = (1-\alpha)\mathbf{x}^n + \alpha F(\mathbf{x}^n)$$ *with* $\alpha \in (0,0.5]$ *for better convergence*

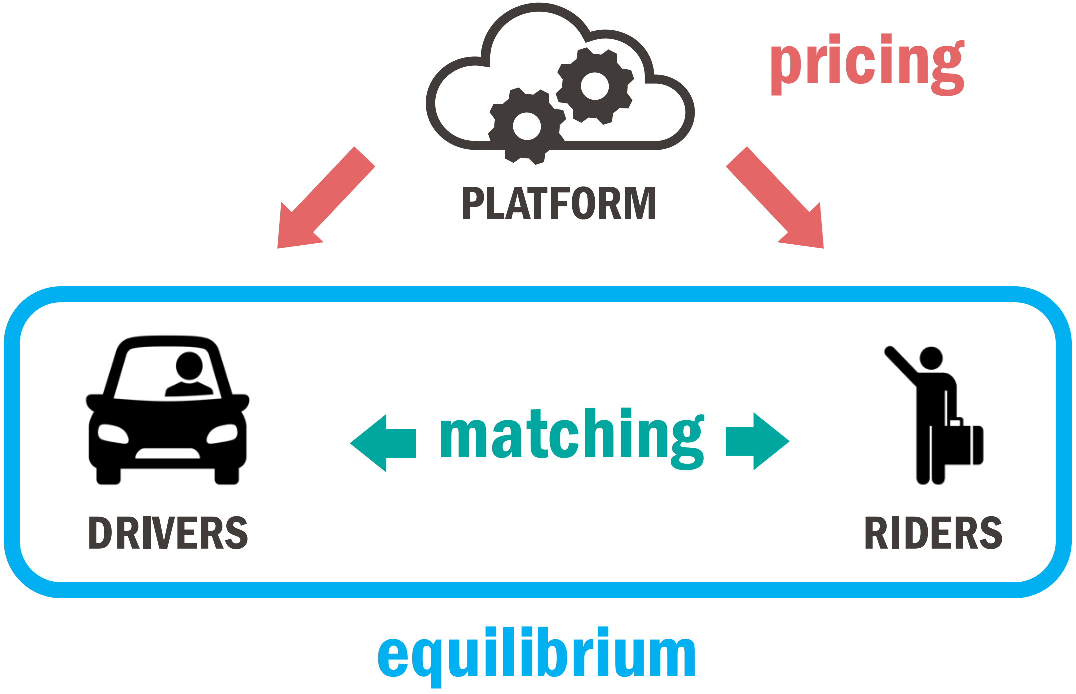
  - Terminate when $||\mathbf{x}^{n+1} - \mathbf{x}^n|| \leq \varepsilon$ for some gap threshold ε

Summary

- Aggregate equilibrium in most two-sided markets can be reduced to a fixed point
- It is then proved to exist by fixed-point theorem and solved via fixed-point iterations

Questions?

Pricing



Pricing

- Optimal pricing problem
  - determine trip fare f and payment rate η to maximize platform profit

$$\max_{f,\eta} \quad R(f,\eta) = (f - \eta\tau)Q(\mathbf{x}^*)$$
$$s.t. \quad \mathbf{x}^* = F(\mathbf{x}^*; f,\eta)$$

- $Q(\mathbf{x}^*) = D(f, w^*)$: demand at equilibrium $\mathbf{x}^* = (w^*, N^*)$

Mathematical Program with Equilibrium Constraints (MPEC)[1]

- *mostly non-linear and non-convex*
- *often solved by sensitivity-based algorithm*

1 Dempe. Annotated bibliography on bilevel programming and mathematical programs with equilibrium constraints. 2003.

Pricing

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- Solve MPEC problem using gradient-based method
  - applicable when equilibrium is expressed by a fixed point and locates in the interior of the feasible set

$$\max_{f,\eta} \quad R(f,\eta) = (f - \eta\tau)Q(\mathbf{x}^*)$$

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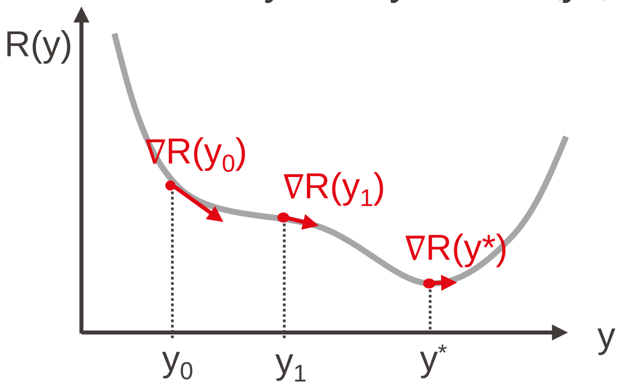
- $Q(\mathbf{x}^*) = D(f, w^*)$: demand at equilibrium $\mathbf{x}^* = (w^*, N^*)$

Let $\mathbf{y} = (f, \eta)$, then the gradient ascent iteration is

$$\mathbf{y}^{n+1} = \mathbf{y}^n + \alpha \nabla R(\mathbf{y}^n)$$

- α: constant step size



Pricing

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Let $\mathbf{y} = (f, \eta)$, then the gradient ascent iteration is

$$\mathbf{y}^{n+1} = \mathbf{y}^n + \alpha \nabla R(\mathbf{y}^n)$$

- α: constant step size

Gradient $\nabla R = \left[\frac{\partial R}{\partial f}, \frac{\partial R}{\partial \eta}\right]^T$ is evaluated as

$$\frac{\partial R}{\partial f} = Q(\mathrm{x}^*) + (f - \eta\tau)\left(\nabla_f Q(\mathrm{x}^*) + \nabla_w Q(\mathrm{x}^*)\frac{\partial w^*}{\partial f}\right)$$

$$\frac{\partial R}{\partial \eta} = -\tau Q(\mathrm{x}^*) + (f - \eta\tau)\nabla_w Q(\mathrm{x}^*)\frac{\partial w^*}{\partial \eta}$$

** Evaluate using current equilibrium*

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Pricing

- Solve MPEC problem using gradient-based meth
  - applicable when equilibrium is expressed by a fixed point and locates in the interior of the feasible set

$$\max_{f,\eta} \quad R(f,\eta) = (f-\eta\tau)Q(\mathbf{x}^*)$$

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$$\frac{\partial R}{\partial \eta} = -\tau Q(\mathrm{x}^*) + (f-\eta\tau)\nabla_w Q(\mathrm{x}^*)\frac{\partial w^*}{\partial \eta}$$

* *Sensitivities of current equilibrium* $\frac{\partial \mathbf{x}^*}{\partial \mathbf{y}} \in \mathbb{R}^{2\times 2}$

Pricing

- Solve equilibrium sensitivity from a linear system

Rewrite the equilibrium as

$$\begin{bmatrix} w^* \\ N^* \end{bmatrix} = \begin{bmatrix} W(w^*, N^*; f, \eta) \\ S(w^*, N^*; f, \eta) \end{bmatrix}$$

Differentiate both sides of the equilibrium yields

$$\frac{\partial w^*}{\partial f} = \nabla_w W(w^*, N^*; f, \eta) \frac{\partial w^*}{\partial f} + \nabla_N W(w^*, N^*; f, \eta) \frac{\partial N^*}{\partial f} + \nabla_f W(w^*, N^*; f, \eta)$$

$$\frac{\partial w^*}{\partial \eta} = \nabla_w W(w^*, N^*; f, \eta) \frac{\partial w^*}{\partial \eta} + \nabla_N W(w^*, N^*; f, \eta) \frac{\partial N^*}{\partial \eta} + \nabla_\eta W(w^*, N^*; f, \eta)$$

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$$\frac{\partial N^*}{\partial \eta} = \nabla_w S(w^*, N^*; f, \eta) \frac{\partial w^*}{\partial \eta} + \nabla_N S(w^*, N^*; f, \eta) \frac{\partial N^*}{\partial \eta} + \nabla_\eta S(w^*, N^*; f, \eta)$$

Rearrange into a linear system

$$\begin{bmatrix} 1 - \nabla_w W & 0 & -\nabla_N W & 0 \\ 0 & 1 - \nabla_w W & 0 & -\nabla_N W \\ -\nabla_w S & 0 & 1 - \nabla_N S & 0 \\ 0 & -\nabla_w S & 0 & 1 - \nabla_N S \end{bmatrix} \begin{bmatrix} \partial w^*/\partial f \\ \partial w^*/\partial \eta \\ \partial N^*/\partial f \\ \partial N^*/\partial \eta \end{bmatrix} = \begin{bmatrix} \nabla_f W \\ \nabla_\eta W \\ \nabla_f S \\ \nabla_\eta S \end{bmatrix}$$

- $(w^*, N^*; f, \eta)$ is omitted for notation simplicity

Pricing

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$$\frac{\partial N^*}{\partial f} = \nabla_w S(w^*, N^*; f, \eta)\frac{\partial w^*}{\partial f} + \nabla_N S(w^*, N^*; f, \eta)\frac{\partial N^*}{\partial f} + \nabla_f S(w^*, N^*; f, \eta)$$

$$\frac{\partial N^*}{\partial \eta} = \nabla_w S(w^*, N^*; f, \eta)\frac{\partial w^*}{\partial \eta} + \nabla_N S(w^*, N^*; f, \eta)\frac{\partial N^*}{\partial \eta} + \nabla_\eta S(w^*, N^*; f, \eta)$$

Rearrange into a linear system

$$(I - A)\frac{\partial \mathbf{x}^*}{\partial \mathbf{y}} = b \quad \Rightarrow \quad \frac{\partial \mathbf{x}^*}{\partial \mathbf{y}} = (I - A)^{-1} b$$

- $A = \begin{bmatrix} \nabla_w W & 0 & \nabla_N W & 0 \\ 0 & \nabla_w W & 0 & \nabla_N W \\ \nabla_w S & 0 & \nabla_N S & 0 \\ 0 & \nabla_w S & 0 & \nabla_N S \end{bmatrix}$
- $b = [\nabla_f W, \nabla_\eta W, \nabla_f S, \nabla_\eta S]^{\mathrm{T}}$

** We do not need to do this by hand but use automatic differentiation*

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Pricing

- Gradient-based algorithm with equilibrium sensitivity
  - Initialize with a feasible solution $\mathbf{y}^0$
  - At each iteration iteration n,
    - Solve market equilibrium $\mathbf{x}^*$ at current solution $\mathbf{y}^n$
    - Compute equilibrium sensitivities $\frac{\partial \mathbf{x}^*}{\partial \mathbf{y}^n}$
    - Evaluate gradient $\nabla R(\mathbf{y}^n)$
    - Update solution by gradient ascent $\mathbf{y}^{n+1} = \mathbf{y}^n + \alpha \nabla R(\mathbf{y}^n)$
  - Terminate when $||\nabla R(\mathbf{y}^n)|| \leq \varepsilon$ for some gap threshold ε

** Similar to all gradient-based algorithms, it only reaches local optimum and thus random initializations are needed to derive global optimum*

Pricing

- Impacts of regulations
  - e.g., min wage and max fleet[1]

$$\max_{f,\eta} \quad R(f,\eta) = (f - \eta\tau)Q(\mathbf{x}^*)$$
$$s.t. \quad \mathbf{x}^* = F(\mathbf{x}^*; f,\eta)$$
$$h(\mathbf{x}^*) \leq 0$$

Reformulation with Lagrangian multiplier

$$\min_{\lambda}\max_{f,\eta} \quad \mathcal{L}(f,\eta,\lambda) = (f - \eta\tau)Q(\mathbf{x}^*) - \lambda h(\mathbf{x}^*)$$
$$s.t. \quad \mathbf{x}^* = F(\mathbf{x}^*; f,\eta)$$

Solution algorithm

- Inner-loop: solve optimal pricing (f^*,η^*) at current multiplier λ^k
- Outer-loop: update multiplier $\lambda^{k+1} = \lambda^k + \rho h(\mathbf{x}^*)$ with constant penalty ρ

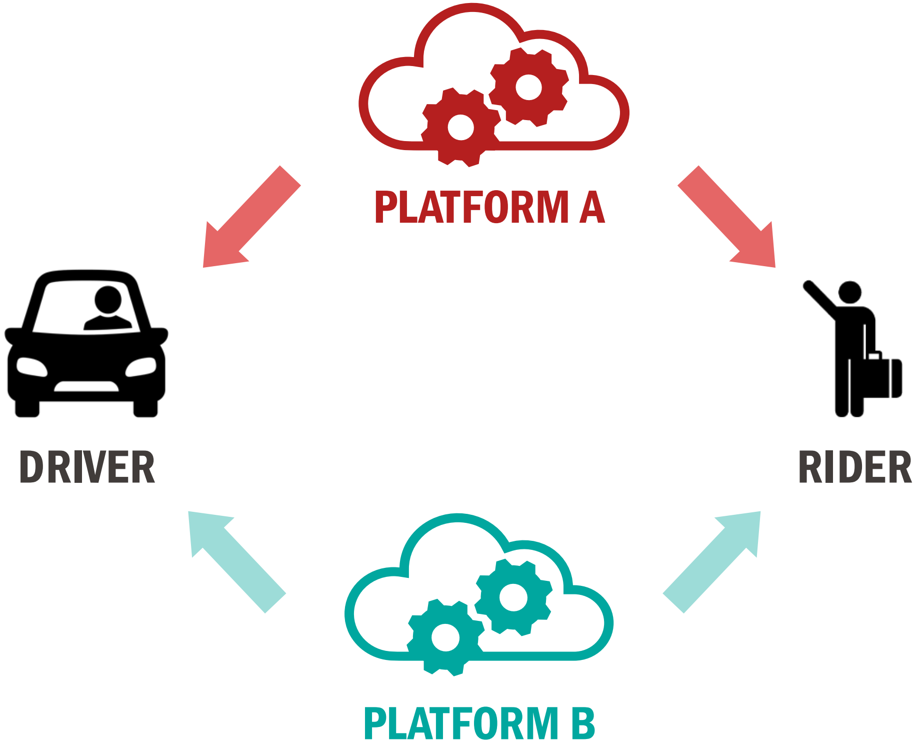
1 Zhang and Nie. To pool or not to pool: Equilibrium, pricing and regulation. 2021.

Summary

- Platform operation is formulated as MPEC and solved by gradient-based algorithm
- A critical step is to solve equilibrium sensitivity by differentiating a fixed point
- Some regulations act as additional constraints

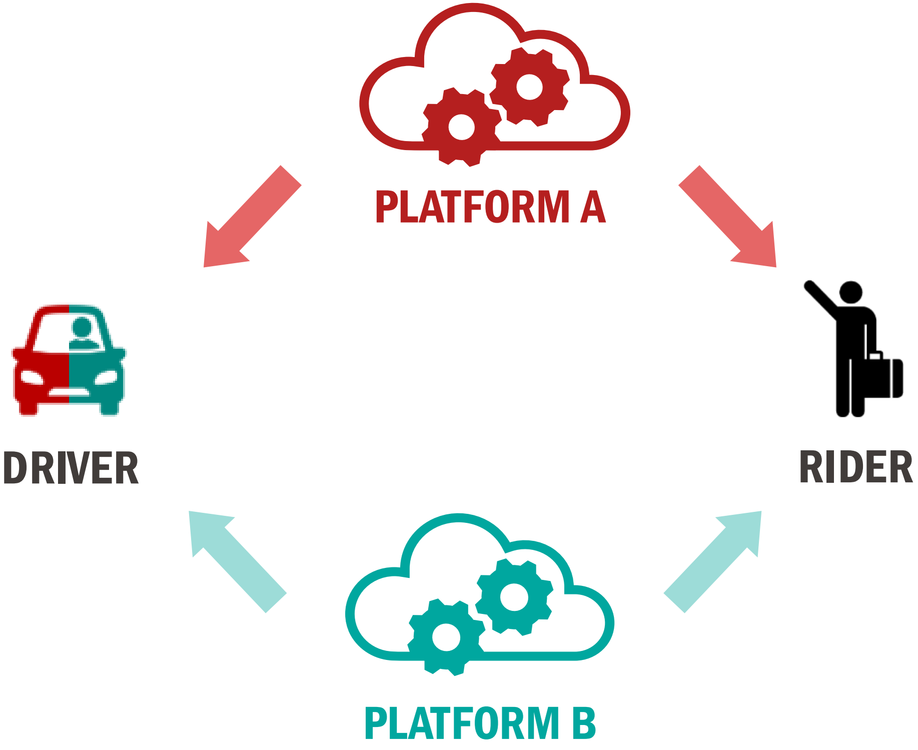
Questions?

Platform competition



Platform competition

- Single-homing vs multi-homing



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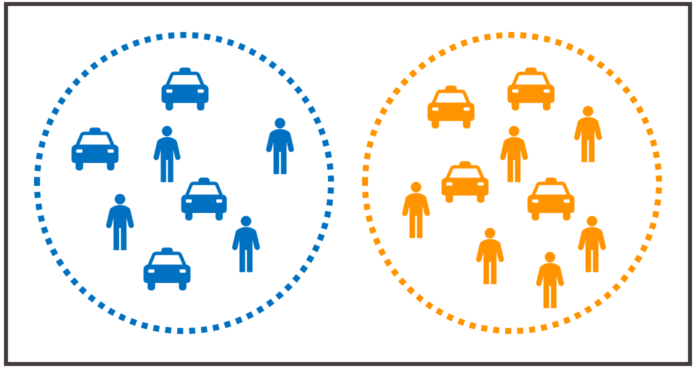
Platform competition

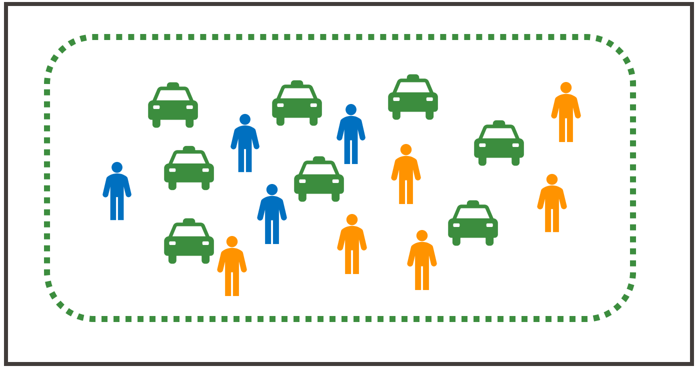
- Single-homing vs multi-homing

single-homing



$$\mathbb{E}[w_i] = \frac{\delta}{2v}\sqrt{\frac{\Pi_i}{\Lambda_i}}$$

multi-homing



$$\mathbb{E}[w_i] = \frac{\delta}{2v}\sqrt{\frac{\sum_j \Pi_j}{\Lambda}}$$

** All unmatched riders $\sum_j \Pi_j$ compete for the same pool of empty vehicles Λ*

Platform competition

- Platform pricing game

Each platform i solves its optimal pricing in anticipation of other platforms' strategies

$$\max_{\mathbf{y}_i} \quad R_i(\mathbf{x}_i^*, \mathbf{y}_i, \mathbf{y}_{-i})$$
$$s.t. \quad \mathbf{x}_i^* = F(\mathbf{x}_i^*; \mathbf{y}_i, \mathbf{y}_{-i})$$

- $\mathbf{x}_i^*$: state variables of platforms i
- $\mathbf{y}_i$: pricing of platform i
- $\mathbf{y}_{-i}$: pricing of platforms other i

Nash equilibrium among platforms is equivalent to VI solution $\mathbf{y}^+$ such that

$$\langle \nabla R(\mathbf{x}^*, \mathbf{y}^+), \mathbf{y} - \mathbf{y}^+ \rangle \leq 0, \forall \mathbf{y} \in \boldsymbol{\Omega}$$

- $\mathbf{y}$: pricing of all platforms $[\mathbf{y}_i]_{\forall i}$
- $\nabla R(\mathbf{x}, \mathbf{y})$: pseudo gradient of platform profit vector $[R_i(\mathbf{x}_i^*, \mathbf{y}_i, \mathbf{y}_{-i})]_{\forall i}$

Existence of equilibrium is proved by evoking theorem of VI solution existence[1]. When it locates in the interior of the feasible set, the similar gradient ascent algorithm for single-platform pricing can be used

$\nabla R(\mathbf{x}^*, \mathbf{y}) = \nabla_{\mathbf{x}^*} R(\mathbf{x}^*, \mathbf{y}) \frac{\partial \mathbf{x}^*}{\partial \mathbf{y}} + \nabla_{\mathbf{y}} R(\mathbf{x}^*, \mathbf{y})$, with $\frac{\partial \mathbf{x}^*}{\partial \mathbf{y}}$ solves linear system $\frac{\partial \mathbf{x}^*}{\partial \mathbf{y}} = \nabla_{\mathbf{x}^*} F(\mathbf{x}^*, \mathbf{y}) \frac{\partial \mathbf{x}^*}{\partial \mathbf{y}} + \nabla_{\mathbf{y}} F(\mathbf{x}^*, \mathbf{y})$

1 Zhang and Nie. Inter-platform competition in a regulated ride-hail market with pooling. 2021.

Summary

- Competition among platform leads to another equilibrium on top of the market equilibrium
- The same gradient-based algorithm is applicable to solve interior equilibrium

Questions?

EPFL

Some findings

- street-hailing vs e-hailing
  - the efficiency of e-hailing in high-density market is overestimated due to the ignorance of passenger competition[1]
- solo vs pooling rides
  - effective pooling helps increase both platform profit and trip throughput[2]
- min wage vs max fleet
  - min wage only improves social welfare in the short-term but can be even harmful in a long run[2]
- single- vs multi-homing
  - multi-homing may induce “tragedy of commons” and lead to insufficient vehicle supply[3]

1 Zhang et al. An efficiency paradox of uberization. 2019.
2 Zhang and Nie. To pool or not to pool: Equilibrium, pricing and regulation. 2021.
3 Zhang and Nie. Inter-platform competition in a regulated ride-hail market with pooling. 2021.

Extend to zone-based model

- zonal movements of drivers
  - searching and charging strategies of profit-max drivers[1,2]
- location-based operations
  - surge pricing and rebalancing[1,3]
- location-based regulations
  - trip-based vs cordon-based congestion fee[4]

1 Zhang et al. RIde-hail vehicle routing (RIVER) as a congestion game. 2023.
2 Zhang and Lygeros. Routing and charging game in ride-hailing service with electric vehicles. 2023
3 Jusup et al. Safe model-based multi-agent mean-field reinforcement learning. 2023.
4 Zhang and Nie. Mitigating traffic congestion induced by transportation network companies: a policy analysis. 2022.

Beyond ride-hailing

- dockless bike-sharing
  - pricing and fleet sizing[1]
  - platform competition with different operational objectives[2]
- meal delivery
  - pros and cons of order bundling[3]
  - mixed fleet of human and AV couriers[4]

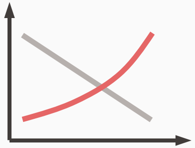
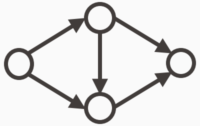
1 Zheng et al. How Many Are Too Many? Analyzing Dockless Bike-Sharing Systems with a Parsimonious Model. 2024.
2 Zheng, Zhang and Nie. Does dockless bikesharing create a competition for losers?. 2024.
3 Ye et al. Modeling and managing an on-demand meal delivery system with order bundling. 2024.
4 Ye et al. Modeling an on-demand meal delivery system with human couriers and autonomous vehicles in a spatial market. 2024

Summary

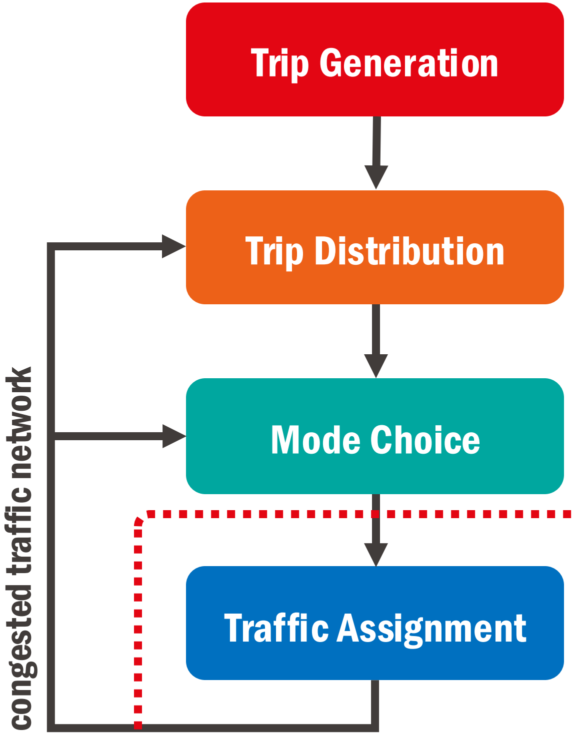
- Aggregate model is a simple yet useful tool to reveal and examine key trade-offs in market equilibrium and service design
- The same modeling framework is easily extended to various transport systems with spatially distributed supply and demand

Questions?

Outline

	Modeling	Optimization	Application
 Aggregate	▪ two-sided market ▪ platform competition	▪ fixed-point iteration ▪ MPEC with fixed point	▪ ride-hailing ▪ bike-sharing ▪ meal delivery
 Network	▪ mixed traffic ▪ multi-modal travel ▪ traffic management	▪ traffic assignment ▪ MPEC with VI	▪ AV routing ▪ MaaS

Four-step model for travel forecasting



trips from/to each zone = F_1(land use, social demographics)

trips for each OD = F_2(zone demand, activities, travel utility)

% trips by mode for each OD = F_3(OD demand, travel time, travel cost)

travel flow on each path/link = F_4(OD demand by mode, network, behavioral principal)

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Traffic assignment

- From route choice to traffic assignment

Route choice

Prob. of individual i choosing route r

$$P_{ir} = h(c_r, \mathbf{c}_{-r}, \beta_i)$$

Network loading

Traffic flow and cost of route r

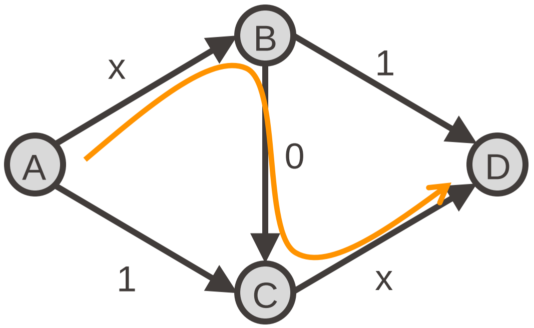
$$f_r = \sum_i P_{ir}$$

$$c_r = \sum_{a \in r} t_a\left(\sum_{a \in r} f_r\right)$$

Traffic assignment

- Behavioral principal
  - user equilibrium (UE): ***selfish*** travelers minimizing ***own*** travel time

Braess network with demand q_{AD}=1



- UE: all travelers take path A-B-C-D with path cost 2

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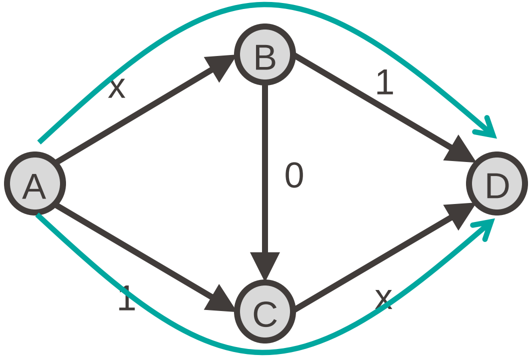
Traffic assignment

- Behavioral principal
  - user equilibrium (UE): ***selfish*** travelers minimizing ***own*** travel time
    - stochastic user equilibrium (SUE): without perfect info or rationality
  - system optimum (SO): ***selfless*** travelers minimizing ***total*** travel time

Braess network with demand $q_{AD}=1$



- UE: all travelers take path A-B-C-D with path cost 2
- SO: travelers split evenly between path A-B-D and A-C-D with path cost 1.5

Traffic assignment

- From UE to Beckmann formulation
  - also widely known as Wardrop equilibrium[1]

Wardrop's first principle (UE)

The travel costs of all used paths are equal, and less or equal than the unused ones. Therefore, no traveler has incentive to deviate from their current path.

- mathematical expression of UE

$$c_{wr} > \mu_w \Rightarrow f_{wr} = 0$$
$$c_{wr} = \mu_w \Rightarrow f_{wr} \geq 0$$

- c_{wr}: cost of path r between OD pair w
- f_{wr}: flow on path r between OD pair w
- μ_w: min path cost between OD pair w

$$\Leftrightarrow \begin{array}{l} c_{wr} \geq \mu_w \\ f_{wr}(c_{wr} - \mu_w) = 0 \end{array}$$

1 Wardrop. Some Theoretical Aspects of Road Traffic Research. 1952

Traffic assignment

- From UE to Beckmann formulation
  - assume path cost is the sum of link cost and link cost function is determined by its own flow

$$c_{wr} = \sum_{a \in r} t_a(x_a) = \sum_a \delta_{ar} t_a(x_a)$$

$$\Leftrightarrow \mathbf{c} = \Delta^T \mathbf{t}(\mathbf{x}) = \Delta^T \mathbf{t}(\Delta \mathbf{f})$$

- c_{wr}: cost of path r between OD pair w
- t_a: cost of link a
- x_a: traffic flow on link a
- δ_{ar}: binary indicator of link a is on path r
- $\mathbf{c}$: vector of path costs
- $\mathbf{t}$: vector of link cost functions
- $\mathbf{x}$: vector of link flows
- $\mathbf{f}$: vector of path flows
- Δ: link-path incidence matrix

  - matrix representation of equilibrium conditions

$$\begin{aligned} \mathbf{c} - \Lambda^T \boldsymbol{\mu} &\geq \mathbf{0} \\ \langle \mathbf{f}, \mathbf{c} - \Lambda^T \boldsymbol{\mu} \rangle &= \mathbf{0} \\ \Lambda \mathbf{f} &= \mathbf{q} \\ \mathbf{f} &\geq \mathbf{0} \end{aligned}$$

- Λ: OD-path incidence matrix
- $\mathbf{q}$: vector of OD demand

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Traffic assignment

- From UE to Beckmann formulation
  - equivalent optimization problem

$$\min_{\mathbf{f}} \quad z(\mathbf{f})$$
$$s.t. \quad \Lambda\mathbf{f} = \mathbf{q}$$
$$\mathbf{f} \geq \mathbf{0}$$

  - find a function $z(\Delta\mathbf{f})$ such that

$$\nabla z(\mathbf{f}) = \mathbf{c} = \Delta^T \mathbf{t}(\mathbf{x})$$

  - Beckmann function

$$z(\mathbf{f}) = Z(\mathbf{x}) = \sum_a \int_0^{x_a} t_a(u)\,\mathrm{d}u$$
$$\frac{\partial z(\mathbf{f})}{\partial f_{wr}} = \sum_a t_a(x_a)\left(\frac{\partial \sum_w \sum_r \delta_{wr} f_{wr}}{\partial f_{wr}}\right)$$
$$= \sum_a \delta_{wr} t_a(x_a) = c_{wr}$$

KKT conditions

$$\nabla z(\mathbf{f}) - \Lambda^T \boldsymbol{\mu} \geq \mathbf{0}$$
$$\langle \mathbf{f}, \nabla z(\mathbf{f}) - \Lambda^T \boldsymbol{\mu} \rangle = \mathbf{0}$$
$$\Lambda\mathbf{f} = \mathbf{q}$$
$$\mathbf{f} \geq \mathbf{0}$$

UE conditions

$$\mathbf{c} - \Lambda^T \boldsymbol{\mu} \geq \mathbf{0}$$
$$\langle \mathbf{f}, \mathbf{c} - \Lambda^T \boldsymbol{\mu} \rangle = \mathbf{0}$$
$$\Lambda\mathbf{f} = \mathbf{q}$$
$$\mathbf{f} \geq \mathbf{0}$$

Traffic assignment

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- Beckmann formulation for UE

$$\begin{aligned} \min_{\mathbf{x}} \quad & Z(\mathbf{x}) \\ s.t. \quad & \Lambda\mathbf{f} = \mathbf{q} \\ & \Delta\mathbf{f} = \mathbf{x} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

- $Z(\mathbf{x}) = \sum_a \int_0^{x_a} t_a(u)\, \mathrm{d}u$

- Optimization problem for SO

$$\begin{aligned} \min_{\mathbf{x}} \quad & TT(\mathbf{x}) \\ s.t. \quad & \Lambda\mathbf{f} = \mathbf{q} \\ & \Delta\mathbf{f} = \mathbf{x} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

- $TT(\mathbf{x}) = \langle \mathbf{t}(\mathbf{x}), \mathbf{x} \rangle = \sum_a t_a(x_a) x_a$

Traffic assignment

- Beckmann formulation for UE

$$\begin{aligned}\min_{\mathbf{x}} \quad & Z(\mathbf{x}) \\ s.t. \quad & \Lambda\mathbf{f} = \mathbf{q} \\ & \Delta\mathbf{f} = \mathbf{x} \\ & \mathbf{x} \geq \mathbf{0}\end{aligned}$$

- $Z(\mathbf{x}) = \sum_a \int_0^{x_a} t_a(u)\, \mathrm{d}u$

- Beckmann formulation for SO

$$\begin{aligned}\min_{\mathbf{x}} \quad & Z'(\mathbf{x}) \\ s.t. \quad & \Lambda\mathbf{f} = \mathbf{q} \\ & \Delta\mathbf{f} = \mathbf{x} \\ & \mathbf{x} \geq \mathbf{0}\end{aligned}$$

- $TT(\mathbf{x}) = \langle \mathbf{t}(\mathbf{x}), \mathbf{x} \rangle = \sum_a t_a(x_a) x_a$
- $Z'(\mathbf{x}) = \sum_a \int_0^{x_a} mt_a(u)\, \mathrm{d}u$

  where $mt_a(x_a) = \frac{\partial t_a(x_a) x_a}{\partial x_a} = t_a(x_a) + t_a'(x_a) x_a$

** SO is achieved if travelers perceive the marginal cost* $mc_{wr} = \sum_a \delta_{wr} mt_a(x_a)$ *as their travel cost*

** Theoretical foundation of marginal pricing* $\tau_a(x_a) = t_a'(x_a) x_a$

Traffic assignment

- Variational inequality (VI) formulation for UE[1]
  - path flow $\mathbf{f}^*$ is UE iff.

$$\langle \mathbf{c}, \mathbf{f} - \mathbf{f}^* \rangle \geq \mathbf{0}, \qquad \forall \mathbf{f} \in \Omega_{\mathbf{f}} = \{\mathbf{f} | \Lambda \mathbf{f} = \mathbf{q}, \mathbf{f} \geq \mathbf{0} |\}$$

$$\Leftrightarrow \langle \Delta^T \mathbf{t}(\mathbf{x}), \mathbf{f} - \mathbf{f}^* \rangle \geq \mathbf{0} \Leftrightarrow \langle \mathbf{t}(\mathbf{x}), \Delta \mathbf{f} - \Delta \mathbf{f}^* \rangle \geq \mathbf{0} \Leftrightarrow \langle \mathbf{t}(\mathbf{x}), \mathbf{x} - \mathbf{x}^* \rangle \geq \mathbf{0}$$

  - link flow $\mathbf{x}^*$ is UE iff.

$$\langle \mathbf{t}(\mathbf{x}), \mathbf{x} - \mathbf{x}^* \rangle \geq \mathbf{0}, \qquad \forall \mathbf{x} \in \Omega_{\mathbf{x}} = \{\mathbf{x} | \Lambda \mathbf{f} = \mathbf{q}, \Delta \mathbf{f} = \mathbf{x}, \mathbf{x} \geq \mathbf{0}\}$$

- VI formulation for SO
  - link flow $\mathbf{x}^*$ is SO iff.

$$\langle \mathbf{mt}(\mathbf{x}), \mathbf{x} - \mathbf{x}^* \rangle \geq \mathbf{0}, \qquad \forall \mathbf{x} \in \Omega_{\mathbf{x}} = \{\mathbf{x} | \Lambda \mathbf{f} = \mathbf{q}, \Delta \mathbf{f} = \mathbf{x}, \mathbf{x} \geq \mathbf{0}\}$$

** We will continue using this formulation due to its compactness*

1 Dafermos. An algorithm for quadratic programming. 1980.

Traffic assignment

- Existence and uniqueness of equilibrium

$$
\begin{aligned}
(\mathrm{P}) \quad \min_{\mathbf{x}} \quad & Z(\mathbf{x}) = \textstyle\sum_a \int_0^{x_a} t_a(u)\, du \\
s.t. \quad & \Lambda\mathbf{f} = \mathbf{q} \\
& \Delta\mathbf{f} = \mathbf{x} \\
& \mathbf{x} \geq \mathbf{0}
\end{aligned}
$$

- when demand and network are properly defined, the feasible set of $\mathbf{x}$ is non-empty, close, and bounded
- when link cost function t_a is continuous, there must exist a solution to P, i.e., equilibrium link flow $\mathbf{x}^*$
- if link cost function t_a is strictly increasing, equilibrium link flow $\mathbf{x}^*$ is unique due to the convexity of P
- Yet, equilibrium path flow $\mathbf{f}^*$ such that $\Delta\mathbf{f}^* = \mathbf{x}^*$ may not be unique

Traffic assignment

- Frank-Wolfe algorithm
  - a solution algorithm for convex program with linear constraints[1]

1 Frank and Wolfe. An algorithm for quadratic programming. 1956.

Traffic assignment

- Frank-Wolfe algorithm
  - a solution algorithm for ***convex*** program with ***linear*** constraints[1]

$$\min_{\mathbf{x}} \quad Z(\mathbf{x}) = \sum_a \int_0^{x_a} t_a(u)\, du$$

convex when $t_a(x_a)$ is monotonically increasing

$$s.t. \quad \Lambda\mathbf{f} = \mathbf{q}, \quad \Delta\mathbf{f} = \mathbf{x}, \quad \mathbf{x} \geq \mathbf{0}$$

linear by definition

  - built upon a linear approximation of objective at a feasible solution $\mathbf{x}_0$

$$\tilde{Z}(\mathbf{x}) = Z(\mathbf{x}_0) + \langle \nabla Z(\mathbf{x}_0), \mathbf{x} - \mathbf{x}_0 \rangle$$

  - this leads to a linear subproblem

$$\min_{\mathbf{x}} \quad \langle \nabla Z(\mathbf{x}_0), \mathbf{x} \rangle \quad s.t. \quad \Lambda\mathbf{f} = \mathbf{q}, \quad \Delta\mathbf{f} = \mathbf{x}, \quad \mathbf{x} \geq \mathbf{0} \quad \Leftrightarrow \quad \min_{\mathbf{f}} \quad \langle \mathbf{c}_0, \mathbf{f} \rangle \quad s.t. \quad \Lambda\mathbf{f} = \mathbf{q}, \quad \mathbf{f} \geq \mathbf{0}$$

** Find a path flow vector that min total travel cost with fixed path costs*

** Assign all flow to the shortest path, i.e., all-or-nothing assignment*

1 Frank and Wolfe. An algorithm for quadratic programming. 1956.

Traffic assignment

- Frank-Wolfe algorithm
  - Initialize with a feasible link flow $\mathbf{x}^0$
    - e.g., all-or-nothing assignment based on free flow travel time
  - At each iteration n,
    - Compute link travel time $\mathbf{t}(\mathbf{x}^n)$
    - Perform all-or-nothing assignment and get corresponding link flow $\mathbf{y}^n$
    - Determine step size α via line search:
      - Let $\mathbf{d}^n = \mathbf{y}^n - \mathbf{x}^n$ and find $\alpha \in [0,1]$ that min $\langle \mathbf{t}(\mathbf{x}^n + \alpha \mathbf{d}^n), \mathbf{d}^n \rangle$
    - Update link flow $\mathbf{x}^{n+1} = \mathbf{x}^n + \alpha \mathbf{d}^n$
    - Compute lower bound $L^n = \tilde{Z}(\mathbf{y}^\mathbf{n})$ and upper bound $U^n = Z(\mathbf{x}^\mathbf{n})$
  - Terminate when $||L^n - U^n|| \leq \varepsilon$ for some gap threshold ε

Summary

- Traffic assignment describes how travel flows distribute on congestible traffic network
  - user equilibrium vs system optimum
- Equilibrium is expressed in different ways
  - complementary conditions
  - convex programs
  - variational inequality
- Existence of equilibrium usually holds, though uniqueness requires additional conditions
- Classic traffic assignment is solved efficiently by Frank-Wolfe algorithm

Questions?

Extend to multiple user classes

- Mixed traffic equilibrium
  - a finite number of user classes $i \in I$ perceive link travel time differently $t_{ia}(x_a)$, where $x_a = \sum_i x_{ia}$ is the total travel flow on link a
  - Beckmann function may no longer exists but VI formulation normally does

$\mathbf{X}^*$ is an equilibrium joint link flow if it is a solution to VI problem

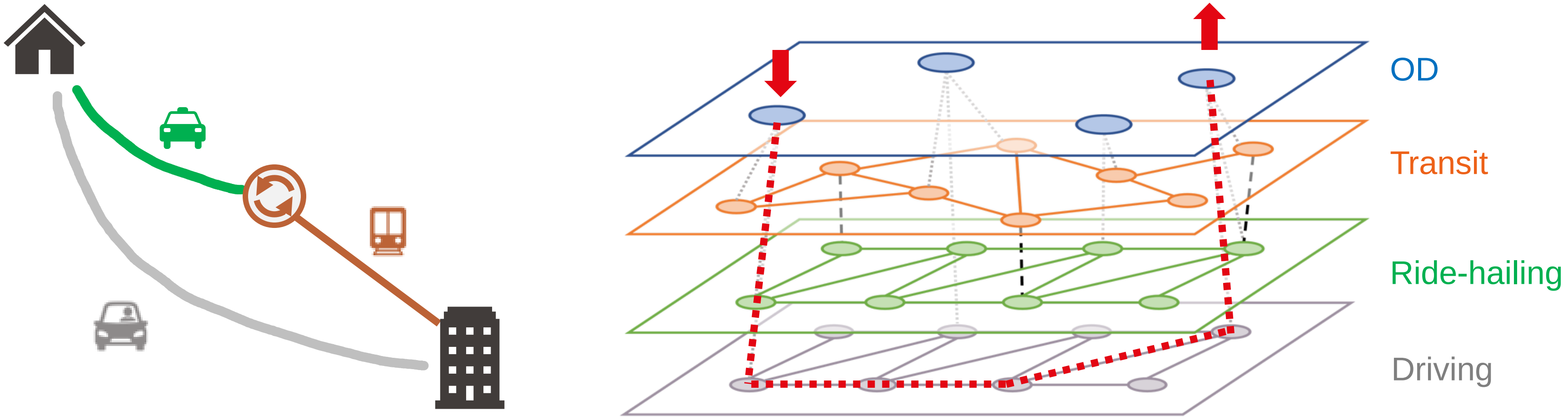
$$\langle \mathbf{T}(\mathbf{x}^*), \mathbf{X} - \mathbf{X}^* \rangle \geq \mathbf{0}, \qquad \forall \mathbf{X} \in \Omega_{\mathbf{X}} = \cup_i \Omega_{\mathbf{x}_i}$$

- $\mathbf{X} = [\mathbf{x}_i]_{\forall i \in I}$: joint link flow
- $\mathbf{T} = (\mathbf{t}_i)_{\forall i \in I}$: joint link cost function
- $\mathbf{x} = \sum_i \mathbf{x}_i = [x_a]_{\forall a}$: aggregate link flow
- $\Omega_{\mathbf{x}_i} = \{\mathbf{x}_i | \Lambda \mathbf{f}_i = \mathbf{q}_i, \Delta \mathbf{f}_i = \mathbf{x}_i, \mathbf{x}_i \geq \mathbf{0}\}$

** While the class-specific equilibrium link flow $\mathbf{X}^*$ is usually non-unique, the aggregate equilibrium link flow $\mathbf{x}^*$ is unique in many applications*

Extend to multiple travel modes

- Multi-modal traffic assignment



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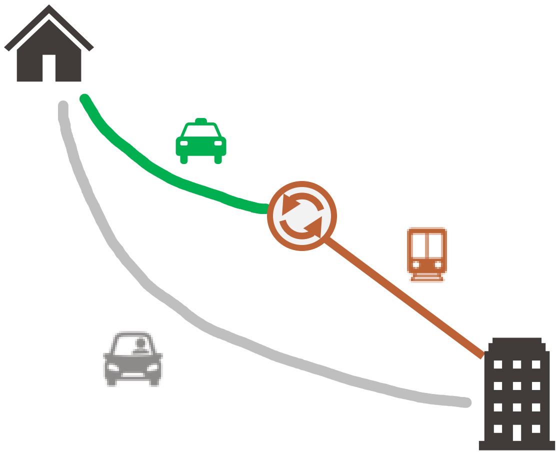
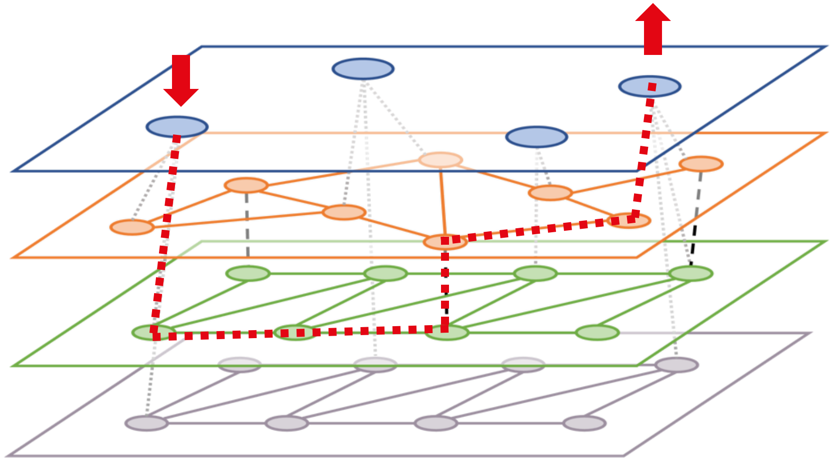
Extend to multiple travel modes

- Multi-modal traffic assignment





- additional links connecting subnetworks
  - e.g., dummy link (zero cost) and transfer link (access cost)
- interactions between subnetworks
  - e.g., ride-hailing and driving share the same road network
- link capacity constraint in some subnetworks
  - e.g., transit links

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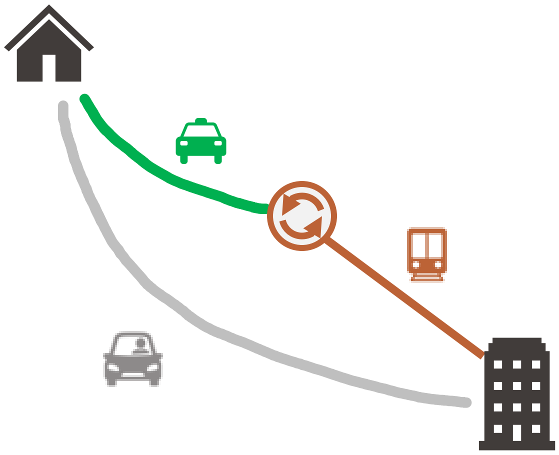
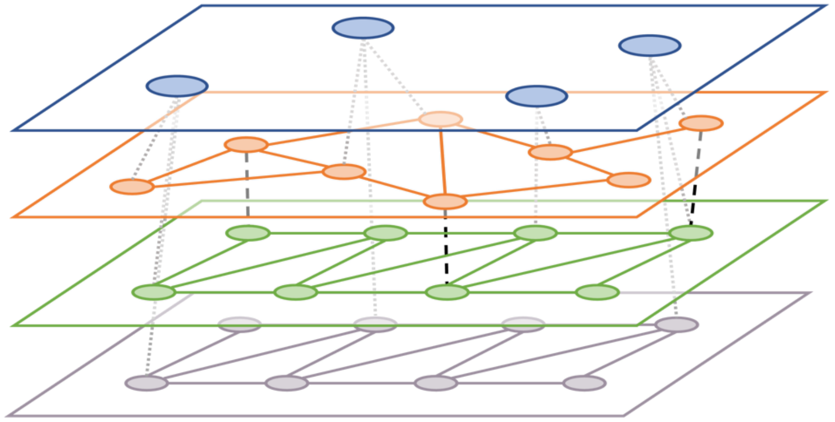
Extend to multiple travel modes

- Multi-modal traffic assignment





- VI formulation still holds in most scenarios, similar to mixed traffic equilibrium

$$\langle \mathbf{T}(\mathbf{x}^*) + \boldsymbol{\lambda}^*, \mathbf{X} - \mathbf{X}^* \rangle \geq \mathbf{0}, \qquad \forall \mathbf{X} \in \Omega_{\mathbf{X}}$$

  - λ^* : Lagrangian associated with capacitated links, also known as ”shadow price”

Traffic management

- Motivation
  - correct the inefficiency of UE compared to SO
  - achieve other objectives (e.g., equity)

- Stackelberg game framework
  - traffic manager as "leader" who makes changes to the traffic network
  - travelers as "follower" who adjust their behaviors in response

- MPEC formulation

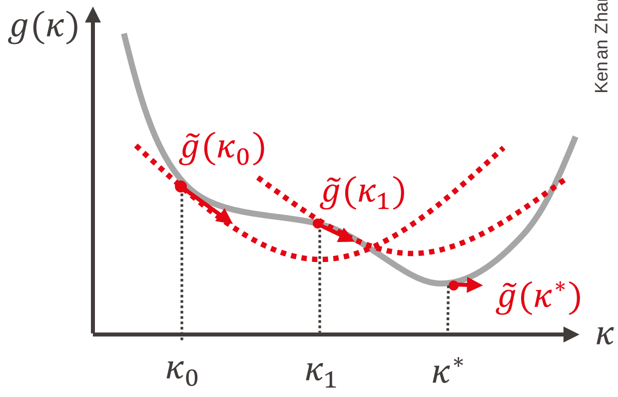
$$\min_{\boldsymbol{\kappa} \in \mathbf{K}} \quad g(\mathbf{x}^*, \boldsymbol{\kappa})$$
$$s.t. \quad \langle \mathbf{t}(\mathbf{x}^*; \boldsymbol{\kappa}), \mathbf{x} - \mathbf{x}^* \rangle \geq \mathbf{0}, \qquad \forall \mathbf{x} \in \Omega_{\mathbf{x}}(\boldsymbol{\kappa})$$

** Share the same structure as MPEC with fixed point but more challenging to solve*

Traffic management

- Reformulation and solved as a bi-level program
  - Upper level:

$$\min_{\boldsymbol{\kappa}} \quad g(\tilde{\mathbf{x}}(\boldsymbol{\kappa}), \boldsymbol{\kappa})$$
$$s.t. \quad \boldsymbol{\kappa} \in \overline{\mathbf{K}}$$



    - approximated equilibrium mapping $\tilde{\mathbf{x}}(\boldsymbol{\kappa}) = \mathbf{x}^*(\boldsymbol{\kappa}_0) + \frac{\partial \mathbf{x}^*(\boldsymbol{\kappa}_0)}{\partial \boldsymbol{\kappa}}(\boldsymbol{\kappa} - \boldsymbol{\kappa}_0)$ based on some known equilibrium $\mathbf{x}^*(\boldsymbol{\kappa}_0)$ and its sensitivity $\frac{\partial \mathbf{x}^*(\boldsymbol{\kappa}_0)}{\partial \boldsymbol{\kappa}}$
    - constrained feasible set $\overline{\mathbf{K}}$ (e.g., a small neighborhood of $\boldsymbol{\kappa}_0$)
  - Lower level:

$$\langle \mathbf{t}(\mathbf{x}; \boldsymbol{\kappa}), \mathbf{x} - \mathbf{x}^* \rangle \geq \mathbf{0}, \qquad \forall \mathbf{x} \in \Omega_{\mathbf{x}}(\boldsymbol{\kappa})$$

Traffic management

- Reformulation and solved as a bi-level program
  - Initialize with some feasible $\boldsymbol{\kappa}^0$
  - At each iteration n,
    - Solve lower-level equilibrium $\mathbf{x}^*(\boldsymbol{\kappa}^n)$
    - Evaluate equilibrium sensitivity $\frac{\partial \mathbf{x}^*(\boldsymbol{\kappa}^n)}{\partial \boldsymbol{\kappa}}$
    - Construct equilibrium mapping $\tilde{\mathbf{x}}(\boldsymbol{\kappa}^n)$ and feasible set $\bar{\mathbf{K}}^n$
    - Solve upper-level problem and set optimal solution to be $\boldsymbol{\kappa}^{n+1}$
  - Terminate when $\left|\left|\boldsymbol{\kappa}^{n+1} - \boldsymbol{\kappa}^n\right|\right| \leq \varepsilon$ for some gap threshold ε

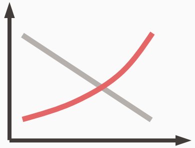
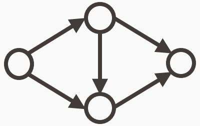
** Highly depend on the problem property and thus customized approaches are often used*

Summary

- Multimodal traffic assignment
  - user class and network structure
- Top-up network design problem

Questions?

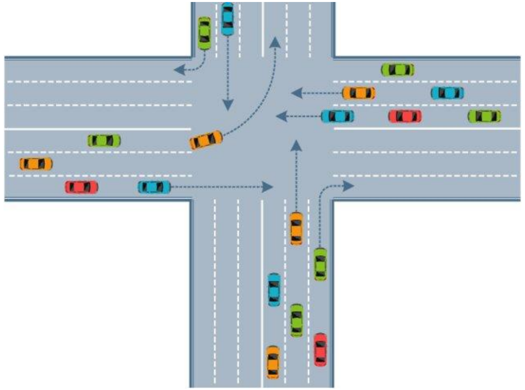
Outline

	Modeling	Optimization	Application
 Aggregate	▪ two-sided market ▪ platform competition	▪ fixed-point iteration ▪ MPEC with fixed point	▪ ride-hailing ▪ micromobility ▪ meal delivery
 Network	▪ mixed traffic ▪ multi-modal travel ▪ traffic management	▪ traffic assignment ▪ MPEC with VI	▪ AV routing ▪ MaaS

AVs in traffic

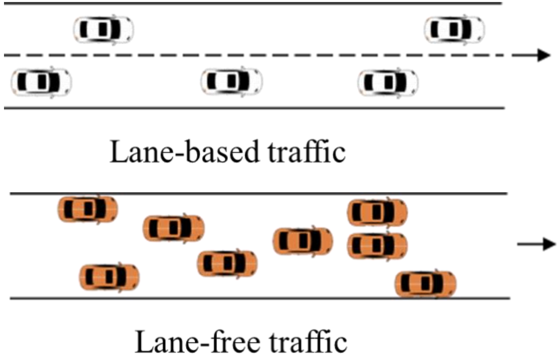
- Microscopic
  - speed harmonization
  - highway platooning
  - signal-free intersection
  - lane-free traffic

CONTROL MOVEMENT



Zhang, Li and Li (2023)

Sugiyama et al. (2008)



TUM DFG Lane Free Traffic

AVs in traffic

- Microscopic
  - speed harmonization
  - highway platooning
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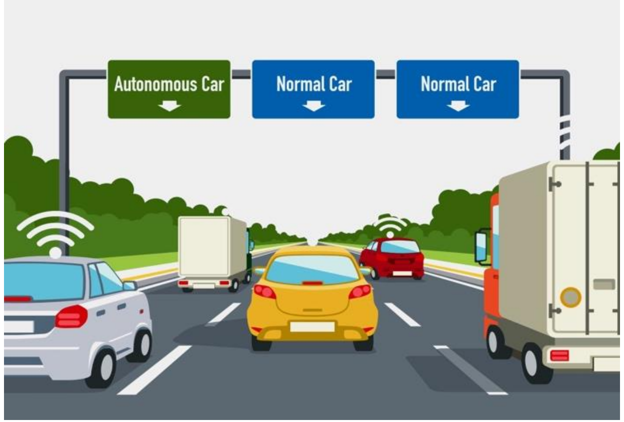
CONTROL MOVEMENT



- Macroscopic
  - dedicated lane
  - route coordination

CONTROL ROUTING

AVs in traffic

- Can we route a fraction of AVs to reduce congestion?
  - Regular vehicles (RVs) and uncontrolled AVs
    - choose route to min own travel time $\Leftrightarrow \arg\min_r \sum_a \delta_{ra} t_a(x_a)$
  - Controlled AVs
    - choose route to min total travel time $\Leftrightarrow \arg\min_r \sum_a \delta_{ra} m t_a(x_a)$

traffic equilibrium with two user classes

- Which and how many AVs should we control?
  - Control AVs by OD pair and bound by total demand $\Leftrightarrow \tilde{\mathbf{q}} \in [\mathbf{0}, \mathbf{q}_{AV}]$
  - Balance control intensity (i.e., # controlled ODs and vehicles) $\Leftrightarrow \min \|\tilde{\mathbf{q}}\|_1$ and system efficiency(i.e., total travel time) $\Leftrightarrow \min TT(\mathbf{x})$

network design problem

AVs in traffic

- Optimal ratio control scheme (ORCS)[1]

$$\min_{\tilde{\mathbf{q}}} \quad \gamma\|\tilde{\mathbf{q}}\|_1 + TT(\mathbf{x}^*)$$
$$s.t. \quad \langle \mathbf{mt}(\tilde{\mathbf{x}}^*), \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^* \rangle \geq \mathbf{0}, \qquad \forall \tilde{\mathbf{x}} \in \Omega_{\tilde{\mathbf{x}}}(\tilde{\mathbf{q}})$$
$$\langle \mathbf{t}(\mathbf{x}^*), \mathbf{x} - \mathbf{x}^* \rangle \geq \mathbf{0}, \qquad \forall \mathbf{x} \in \Omega_{\mathbf{x}}(\mathbf{Q} - \tilde{\mathbf{q}})$$
$$\mathbf{0} \leq \tilde{\mathbf{q}} \leq \mathbf{q}_{AV}$$

- γ: weight of objectives
- $\mathbf{Q}$: total demand

- Bi-level formulation
  - Upper-level:

$$\min_{\boldsymbol{\delta}_{\tilde{\mathbf{q}}}} \quad \gamma\left\|\boldsymbol{\delta}_{\tilde{\mathbf{q}}}\right\|_1 + \left\langle \nabla_{\tilde{\mathbf{q}}} TT\left(\mathbf{x}^*(\tilde{\mathbf{q}}_0)\right), \boldsymbol{\delta}_{\tilde{\mathbf{q}}} \right\rangle$$
$$s.t. \quad \underline{\boldsymbol{\delta}} \leq \boldsymbol{\delta}_{\tilde{\mathbf{q}}} \leq \overline{\boldsymbol{\delta}}$$

- $\boldsymbol{\delta}_{\tilde{\mathbf{q}}}$: additional demand shift
- $\tilde{\mathbf{q}}_0$: current demand shift
- $\overline{\boldsymbol{\delta}}, \underline{\boldsymbol{\delta}}$: upper and lower bound

  - Lower-level:

$$\langle \mathbf{mt}(\tilde{\mathbf{x}}^*), \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^* \rangle \geq \mathbf{0}, \quad \forall \tilde{\mathbf{x}} \in \Omega_{\tilde{\mathbf{x}}}\left(\tilde{\mathbf{q}}_0 + \boldsymbol{\delta}^*_{\tilde{\mathbf{q}}}\right)$$
$$\langle \mathbf{t}(\mathbf{x}^*), \mathbf{x} - \mathbf{x}^* \rangle \geq \mathbf{0}, \qquad \forall \mathbf{x} \in \Omega_{\mathbf{x}}(\boldsymbol{Q} - \tilde{\mathbf{q}}_0 - \boldsymbol{\delta}^*_{\tilde{\mathbf{q}}})$$

1 Zhang and Nie. Mitigating the impact of selfish routing: An optimal-ratio control scheme (ORCS) inspired by autonomous driving. 2018.

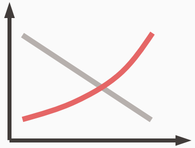
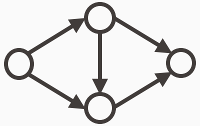
AVs in traffic

- Key findings of ORCS[1]
  - SO can be closely approached by controlling around 10% of all vehicles
    - Does the same result hold in general networks?
  - A small fractions of OD pairs are fully controlled while others are not controlled at all
    - The spatially uneven control leads to equity issue. How to compensate the controlled travelers?

1 Zhang and Nie. Mitigating the impact of selfish routing: An optimal-ratio control scheme (ORCS) inspired by autonomous driving. 2018.

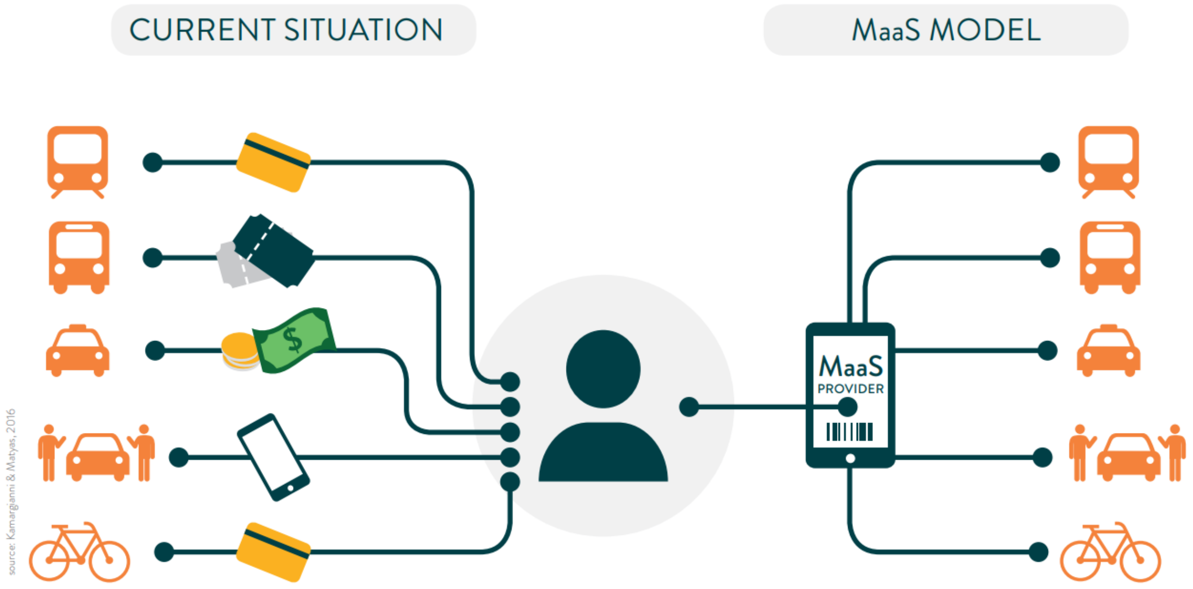
Outline

Kenan Zhang

	Modeling	Optimization	Application
 Aggregate	▪ two-sided market ▪ platform competition	▪ fixed-point iteration ▪ MPEC with fixed point	▪ ride-hailing ▪ micromobility ▪ meal delivery
 Network	▪ mixed traffic ▪ multi-modal travel ▪ traffic management	▪ traffic assignment ▪ MPEC with VI	▪ AV routing ▪ MaaS

Multi-modal travel

- What is Mobility-as-a-Service (MaaS)?
  - Integrates various transport services into a single on-demand mobility service through a single application and payment channel[1]



1 European MaaS Alliance

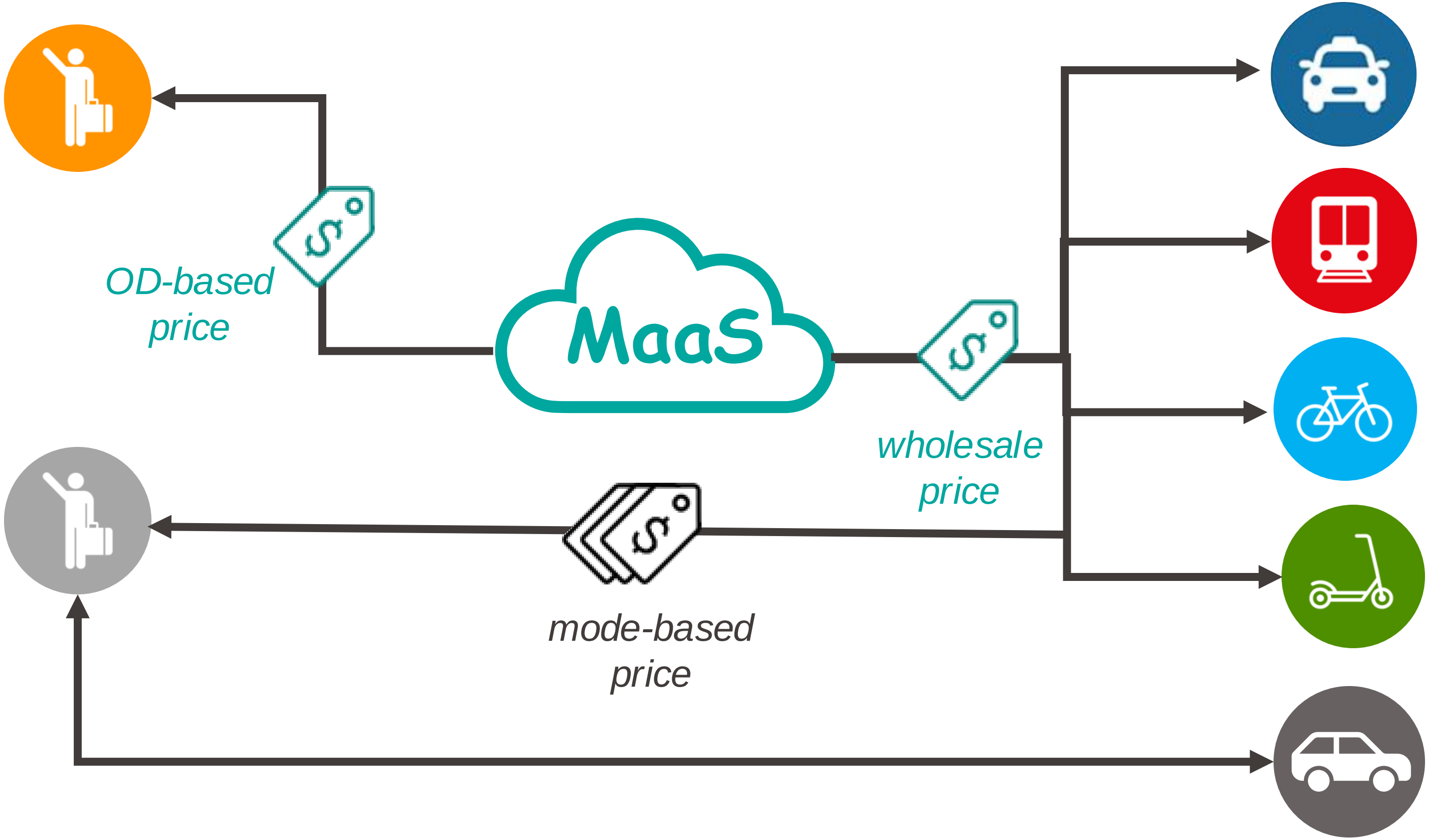
Multi-modal travel

- What is Mobility-as-a-Service (MaaS)?
  - Integrates various transport services into a single on-demand mobility service through a single application and payment channel[1]

- What is role of a MaaS platform?
  - Match travel demand with service capacity on a multi-modal network
  - Negotiate with service providers and price the MaaS trips

1 European MaaS Alliance

Multi-modal travel



Multi-modal travel

- MaaS assignment[1]

  - MaaS and non-MaaS travelers interact in the same multi-modal transportation network

traffic equilibrium with two user classes

  - MaaS platform decides how many MaaS travelers to serve and how much service capacity to purchase

network design problem

1 Yao and Zhang, Design an intermediary mobility-as-a-service (MaaS) platform using many-to-many stable matching framework. 2024

Multi-modal travel

Kenan Zhang

- MaaS assignment*

$$\min_{\mathbf{q},\mathbf{k}} \quad TT(\mathbf{x}^*, \tilde{\mathbf{x}}^*)$$
$$s.t. \quad \langle \mathbf{t}(\mathbf{x}^*), \mathbf{x} - \mathbf{x}^* \rangle \geq \mathbf{0}, \forall \mathbf{x} \in \Omega_{\mathbf{x}}(\mathbf{q})$$
$$\langle \tilde{\mathbf{t}}(\tilde{\mathbf{x}}^*), \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^* \rangle \geq \mathbf{0}, \forall \tilde{\mathbf{x}} \in \Omega_{\tilde{\mathbf{x}}}(\tilde{\mathbf{q}})$$
$$\mathbf{q} + \tilde{\mathbf{q}} = \mathbf{Q},$$
$$\mathbf{x}^* \leq \mathbf{k}, \tilde{\mathbf{x}}^* \leq \mathbf{K} - \mathbf{k}.$$

- $\mathbf{x}, \tilde{\mathbf{x}}$: MaaS and non-MaaS link flow
- $\mathbf{q}, \tilde{\mathbf{q}}$: MaaS and non-MaaS demand
- $\mathbf{t}, \tilde{\mathbf{t}}$: MaaS and non-MaaS link cost
- $\mathbf{Q}$: total demand
- $\mathbf{k}$: MaaS service capacity
- $\mathbf{K}$: total link capacity

* A simplified formulation of Yao and Zhang (2024)

Multi-modal travel

- MaaS assignment*

$$\max_{\boldsymbol{\lambda}} \min_{\mathbf{q}} \quad TT(\mathbf{x}^*, \tilde{\mathbf{x}}^*)$$

$$s.t. \quad \langle \mathbf{t}(\mathbf{x}^*) + \boldsymbol{\lambda}, \mathbf{x} - \mathbf{x}^* \rangle \geq \mathbf{0}, \forall \mathbf{x} \in \Omega_{\mathbf{x}}(\mathbf{q})$$

$$\langle \tilde{\mathbf{t}}(\tilde{\mathbf{x}}^*) + \boldsymbol{\lambda}, \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^* \rangle \geq \mathbf{0}, \forall \tilde{\mathbf{x}} \in \Omega_{\tilde{\mathbf{x}}}(\tilde{\mathbf{q}})$$

$$\mathbf{q} + \tilde{\mathbf{q}} = \mathbf{Q},$$

$$\mathbf{x}^* + \tilde{\mathbf{x}}^* \leq \mathbf{K},$$

$$\boldsymbol{\lambda}(\mathbf{x}^* + \tilde{\mathbf{x}}^* - \mathbf{K}) = \mathbf{0}.$$

- $\mathbf{x}, \tilde{\mathbf{x}}$: MaaS and non-MaaS link flow
- $\mathbf{q}, \tilde{\mathbf{q}}$: MaaS and non-MaaS demand
- $\mathbf{t}, \tilde{\mathbf{t}}$: MaaS and non-MaaS link cost
- $\mathbf{Q}$: total demand
- $\mathbf{k}$: MaaS service capacity
- $\mathbf{K}$: total link capacity
- $\boldsymbol{\lambda}$: Lagrangian multiplier

- Gradient-based algorithm joint with multiplier update
  - solve equilibrium $(\mathbf{x}^*, \tilde{\mathbf{x}}^*)$ and evaluate equilibrium sensitivity $\frac{\partial \mathbf{x}^*}{\partial \mathbf{q}}, \frac{\partial \tilde{\mathbf{x}}^*}{\partial \mathbf{q}}$
  - construct gradient $\nabla_{\mathbf{q}} TT(\mathbf{x}^*, \tilde{\mathbf{x}}^*)$ and perform gradient descent
  - update Lagrangian multiplier $\boldsymbol{\lambda}$ based on constraint violation

* A simplified formulation of Yao and Zhang (2024)

Multi-modal travel

- MaaS assignment[1]

  - MaaS and non-MaaS travelers interact in the same multi-modal transportation network

traffic equilibrium with two user classes

  - MaaS platform decides how many MaaS travelers to serve and how much service capacity to purchase

network design problem

** This is half of the story because the platform needs to properly decide on trip fare and capacity purchase price to achieve the desired MaaS assignment*

1 Yao and Zhang, Design an intermediary mobility-as-a-service (MaaS) platform using many-to-many stable matching framework. 2024

Multi-modal travel

- Key findings of MaaS[1]
  - The launch of MaaS platform can benefit all stakeholders
    - How do service providers respond in terms of their operational strategies?
  - MaaS promote multi-modal travel while reducing private driving
    - What are the prerequisites (e.g., connectivity of public transport networks) to reach this result?

1 Yao and Zhang, Design an intermediary mobility-as-a-service (MaaS) platform using many-to-many stable matching framework. 2024

Summary

- Examples of how mixed traffic equilibrium and network design problem serve as the modeling framework to study AVs and MaaS
- More emerging mobility problems can be framed and optimized in a similar way

Questions?

Connection to behavioral modeling

Kenan Zhang

- So far we have been focusing on the supply side while simplifying or even ignoring many behavioral factors, e.g.,
  - mode and route choice
  - imperfect info and rationality
  - personal preference and characteristics
- Models that capture these factors are obviously ideal but
  - additional challenge in solution procedure
  - introduce more uncertainties that may blur key trade-offs

Thanks!
Q & A

HOMES @ EPFL